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A PHYSICAL STUDY OF SPHERICAL ABERRATION*

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ABSTRACT. Analytical expressions for the intensity of light near the focus of a lens system are usually too complex for satisfactory physical interpretation, especially when the effects of aberration are introduced. This does not diminish the necessity for an accurate knowledge of such effects. It is their experimental determination which is the main subject of the present paper.

PART I deals with the theoretical basis of possible methods for numerical computation on Huygens' principle. Aberration must be expressed in terms of relative phase. Effects at the paraxial focus differ considerably from those at the best visual focus when aberration is present. A short discussion of spherical aberration on the Fresnel zone theory leads to some of the general effects verified in practice.

PART II. Experimental Study. The aberrations of a known microscope objective were computed for varying "tube lengths" and were experimentally checked by the results of a specially devised modification of the Hartmann method. An ultra-microscopic pinhole in a silver film was used as an object. The appearances at and near the focus with differing known aberration residuals were photographed and thus examined for size and light distribution. Visual "extinction" methods were also employed. Curves and photographs illustrate the results.

The "spurious disc" appearance is shown to persist near the best visual focus even in the presence of large amounts of aberration. The effect of increasing aberration is to draw light from the central concentration and scatter it in the surrounding field, and, from measurements of the loss, the necessity of restricting the phase residuals to within $\lambda/6$ is inferred. Spherical aberration is found to produce marked asymmetry on each side of the focus, a matter of great importance in some optical instruments.

* Portion of a Thesis approved for the Degree of Doctor of Science in the University of London.

PART I. THEORETICAL STUDY

ALTHOUGH the physical effect of spherical aberration is a subject of great importance in the study of optical instruments, it has received comparatively little attention. The simplest problem of investigating the diffraction of the circular apertures of aplanatic telescope objectives with light from a "point" source at an infinite distance has of course been thoroughly worked out according to Huygens' principle by Airy*, and others (Lommel† treating the diffraction of a circular aperture without a lens), while the discussion has been extended to cover the case of a finite source of light by Rayleigh‡, Andre§, Struve‖ and Lommel¶; also quite recently H. Nagaoka** has given full numerical results for a number of cases. Strehl†† and Walker‡‡ have given theoretical expressions to represent diffraction phenomena in the presence of spherical aberration, and Lord Rayleigh§§ gave a numerical discussion of the effects of certain asymmetrical aberrations, the results of which paper are well known. More recently Silberstein‖‖ has investigated the diffraction pattern produced by a plano-convex lens, and lastly Prof. Conrady¶¶ and A. Buxton*** have given numerical results obtained (by mechanical integration) for the intensity of light in the diffraction pattern produced by convergent light waves subject to spherical aberration which follows simple laws.

The nature of the distribution of light round the axial focus of a lens is a problem which has frequently been attacked mathematically. It has been quite successfully solved for points in the focal plane of an "aplanatic" lens when the intensity is found to depend on functions of the form (assuming Huygens' principle)

$$I = \frac{a_1^2}{\kappa^2} \mathcal{J}_1^2 \left(\frac{2\pi\kappa a_1}{\lambda R} \right) \qquad \ldots\ldots(1),$$

where $\mathcal{J}_1$ is the Bessel function of the first order, R is the radius of the wave surface at the lens, a_1 the semi-aperture, k the distance of the reference point from the axis, and λ the wave length. At present it has not been found possible to give an expression for the light distribution in the presence of spherical aberration (or away from the focus) which can be physically interpreted in such a simple way. Prof. Conrady has been able, however, to determine the distribution numerically in a series of simple cases by mechanical quadratures which give the value of the expression

$$I = \text{const.} \times \left[\left\{ \int_{a=0}^{a=a_1} \mathcal{J}_0 (\alpha a) \cos \eta \, d(a^2) \right\}^2 + \left\{ \int_{a=0}^{a=a_1} \mathcal{J}_0 (\alpha a) \sin \eta \, d(a^2) \right\}^2 \right], \qquad \left(\text{where } \alpha = \frac{2\pi\kappa}{\lambda R} \right) \quad \ldots\ldots(2),$$

* *Camb. Phil. Trans.* vol. 5.
† *Abhand. Bayer. Akad.* 15 (1884).
‡ "Wave theory of light," *Enc. Britt.* 9th edition.
§ *Ann. l'Ecole Norm.* Sup. 5 (1876); 10 (1881).
‖ *Mém. l'Acad. St Pétersbourg* 30 (1882).
¶ *Abhand. Bayer. Akad.* 19 (1897).
** Nagaoka, *Astrophys. Journ.* vol. 51, no. 2.
†† *Theorie des Fernrohrs*, Barth, Leipzig (1894).
‡‡ *Proc. Phys. Soc. Lond.* 24 (1912) 160–164.
§§ *Phil. Mag.* 8 (1879) 403.
‖‖ *Ibid.* 35, no. 205 (1918) 30.
¶¶ *Monthly Notices, R.A.S.* vol. 79, no. 8.
*** *Ibid.* vol. 81, no. 8.

η representing the phase, which is supposed to vary in a spherical reference surface (touching the lens) with the particular point at its centre by an amount depending on the spherical aberration. No analytical expression for η can be obtained for any but the simplest optical systems, but empirical expressions can easily be derived from the result of trigonometrical ray tracing or Hartmann tests in the most complex systems, thus enabling the truth of the numerical results for light distribution to be checked experimentally.

It is not too much to say that the thorough solution of the problem is of the greatest importance in the study of the performance of optical instruments, and experimental investigation is necessary both to prepare the way for full numerical work and to assess its likely value as applied to practice.

Relations between methods of expressing aberration.

The above expression can be easily integrated by quadratures, taking each term separately, if η (for successive zones of the surface) can be expressed in terms of a^2. If the integration is performed with a^2 as the variable each elementary zone will contribute an equal amount of light (being of equal area), to the sum. Now spherical aberration may be expressed in terms of axial intercepts between the crossing of rays, lateral intercepts in some plane perpendicular to the axis at a definite position, or in terms of the angle between ideal and actual ray paths. Again aberration is often expressed in terms of phase differences between disturbances from various parts of the aperture at certain positions.

In Fig. 1 the broken line *OB* represents the trace of a surface with constant

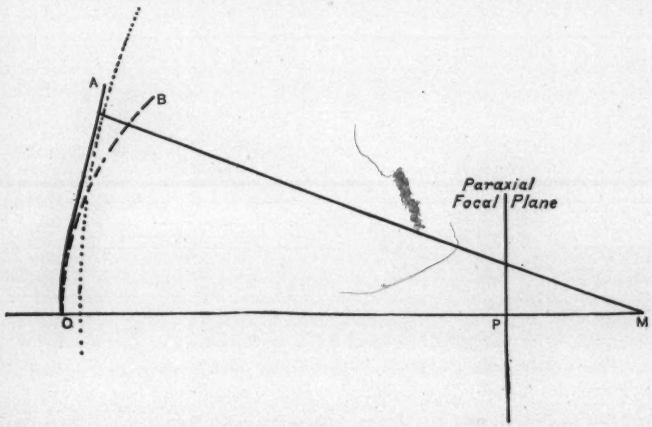


Fig. 1

curvature. Only the semi-aperture is shown. OA represents the trace of some wave front with the same curvature at the axis as that of OB, but decreasing away from the axis. The normals BP and AM at the same aperture include an angle θ. If D is the distance between the wave fronts at a height y we have (very nearly for small apertures)

$$\theta = \frac{dD}{dy} = \frac{\lambda}{2\pi}\frac{d\eta}{dy} \quad \ldots.(3), \quad \text{and, hence,} \quad \eta_y = \frac{2\pi}{\lambda}\int_0^y \theta\, dy^* \quad \ldots.(4),$$

where η represents the phase difference between disturbances from A and B arriving at the "paraxial focus" P, the centre of curvature of OB. The angular aberration θ corresponds to a lateral aberration in the plane through P at right angles to OP,

$$L_p = \theta f$$

very nearly if the aperture is small, where $f = OP$.

This again corresponds to an axial aberration

$$A_p = \frac{L_p \cdot f}{y} = \frac{\theta f^2}{y}.$$

We may write two new expressions for η

$$\eta = \frac{2\pi}{\lambda}\int_0^y \frac{L_p}{f}\, dy = \frac{2\pi}{\lambda}\int_0^y \frac{A_p y}{f^2}\, .\, dy \quad \ldots.(5).$$

These expressions enable us to employ the "geometrical" ray aberrations, found in optical systems by any trigonometrical or practical method, to compute the aberration in terms of phase differences. For another focus at a distance m from P, the new lateral aberration

$$L = L_p - \frac{my}{f} \quad \text{and} \quad A = A_p - m.$$

Therefore
$$\eta = -\frac{\pi m}{\lambda}\cdot\frac{y^2}{f^2} + \frac{2\pi}{\lambda f}\int_0^y L_p\, dy = \frac{2\pi}{\lambda f}\left(\int_0^y L_p\, dy - \frac{my^2}{2f}\right) \quad \ldots.(6)$$

$$= -\frac{\pi m}{\lambda}\cdot\frac{y^2}{f^2} + \frac{2\pi}{\lambda f^2}\int_0^y A_p y\, dy \quad \ldots.(7),$$

both equations being close approximations. We should notice that for symmetrical systems η is a function of even powers only of the aperture y. From the inspection of equation (6) (and otherwise obviously) a term in y^2 corresponds to a change of focus (considered as a first approximation).

The usual analytical expressions for (axial) spherical aberration exhibit it as roughly proportional to the square of the aperture, which indicates in conjunction with equation (7) that the aberration in phase differences varies as the fourth power of the aperture.

Empirical expressions for aberration.

Having thus obtained the phase difference in terms of aperture, the original expression for intensity in the diffraction pattern at any point near the axis can

* The possibility of integrating thus to obtain optical path differences has of course been recognised since early times in the history of optics, but these methods have received little attention.

be numerically integrated. It is well known, however, that the analytical results for aberrations of the third order yield figures which are far from the truth when applied to large aperture systems, and to obtain the phase differences for an *actual* system it is best to calculate them directly from the results of trigonometric ray tracing, carried out by the usual so-called Seidel* equations, if the curves and refractive indices are known. About four rays between centre and margin would give sufficient points for a curve showing the required phase differences and the phase calculation would be most easily performed by the formula given by Prof. Conrady†,

$$W = \Sigma \frac{N' Y \sin I' \sin \frac{1}{2}(U - U') \sin \frac{1}{2}(I - U')}{2 \cos \frac{1}{2}U \cos \frac{1}{2}I \cos \frac{1}{2}(U + I) \cos \frac{1}{2}I' \cos \frac{1}{2}U'} \quad \ldots.(8),$$

(the summation to be carried out for all surfaces of the system), in which I and I' are the angles of incidence and refraction respectively, U and U' the angles made by the incident and refracted beams with the axis, N' the refractive index of the medium in which the refracted ray is found, and Y the distance from the axis. W represents the value of (axial path—special ray path) at the focus of the zone given by Y. If the phase difference at the paraxial focus is required it will therefore be necessary to calculate the position of this latter when the required value may be found from equation (7). On the other hand the calculation of a few rays through a lens will generally render it possible easily to calculate an empirical formula for the aberration‡ which can be directly interpreted to give the phase differences.

For example, a particular type of Galilean binocular objective (focal length = 8 ins.) gave an axial aberration expressed sufficiently nearly up to the full aperture by the formula

$$\text{Axial aberration in inches} = -\cdot0001\, Y^2 + \cdot0267\, Y^4 \quad \ldots.(9).$$

Such a formula for the axial aberration referred to the paraxial focus can readily be used for computing the phase differences at any other axial focus (near by) provided its position relatively to the paraxial focus is known. It will be noticed that the lens has been computed to make the aberration very small according to the usual "third order" analytical theory. In the above case we may compute the phase difference for the margin for $y = 1$ and $f = 8$. It amounts to $3{\cdot}5\lambda$. By bending the surface of separation in the direction of greater curvature we may get a modified aberration

$$A = -0{\cdot}0232y^2 + \cdot0236y^4 \quad \ldots.(10).$$

The axial aberration is now almost zero for the marginal zone and the phase difference is reduced to $1{\cdot}6\lambda$. In either case, however, it is possible to choose another focus at which the residual aberration is materially lessened. It might be noted that the whole aperture of a Galilean binocular objective is not used for any one point of the image, the entrance pupil depending on the aperture of the eye pupil when the whole refracting system in front of the retina is considered.

* These are really due to Bessel and were published by him in 1841.

† *Monthly Notices, R.A.S.* Apr. 1905.

‡ Reference may also be made to a paper by Baker and Filon, *Trans. Opt. Soc.* 20 (1918–19) 67.

Effect of a change of focus.

The possibility of choosing a better focus may be more fully appreciated by making use of a graphical device which was mentioned by S. D. Chalmers*.

Referring to equation (5) it is evident that if we plot a curve connecting lateral aberration (x) with aperture (y) and find the area included between it and the 'y' axis, then on multiplying by $\frac{2\pi}{\lambda f}$ we have the phase difference with respect to the marginal focus. It is also apparent from (6) that the phase difference with respect to another focus at a distance m will be found from the area of the curve *with respect to a line drawn through the origin*, given by

$$x = \frac{y \cdot m}{f},$$

the whole being multiplied by $\frac{2\pi}{\lambda f}$ as before.

Referring now to Figs. 2 and 3, they respectively represent a curve similar to that corresponding to (9) and a curve similar to that given by equation (10). It is seen at once that the residual aberrations with respect to the foci given by *Ob* and *Oc* respectively will be considerably less than the aberrations at the paraxial focus.

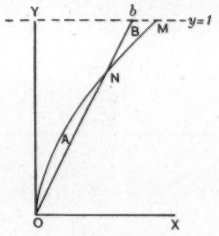


Fig. 2

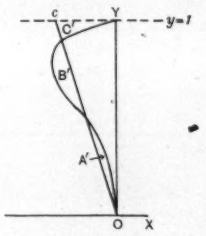


Fig. 3

The residuals are a minimum in the first case where the areas A and B are equal and in the second case where the two largest of the areas A', B', C' are equal and opposite in sign. In Prof. Conrady's paper on "Star Discs" the magnitudes of the residual aberrations for these and a number of similar cases are worked out, and it is shown that in dealing with simple spherical aberration of the type given by

$$\eta = \text{const.} \times y^4,$$

the residual phase difference at the best focus is only a quarter of the maximum phase difference at the marginal or paraxial foci, this best focus lying half way between them. Other cases of greater complexity are dealt with in a similar manner.

* "On the Calculation of Telescope Objectives free from Aberrations," *Trans. Opt. Soc.* 18, no. 4. Strehl has worked out many of these graphical methods.

Phase residuals in actual systems.

In the paper by Chalmers, to which previous reference was made, the results of tests carried out by Hartmann are discussed. One lens in question was a photographic (astronomical) objective of semi-aperture 40 mm. and focal length 1 metre. It is evident from the curve (Fig. 4), as Chalmers suggests, that the objective was originally designed to bring the marginal ray to the paraxial focus (called the "theoretical focus" in the paper), when the aberrations would have been represented approximately by the curve of which the top portion is dotted in. Computing (from the area of the curve) the phase difference of the marginal ray at the paraxial focus in this imagined case it is found to be very nearly $1{\cdot}0\lambda$. On going to a focus ·6 mm. nearer the objective, however, we could reduce the residual phase aberration to $\lambda/4$.

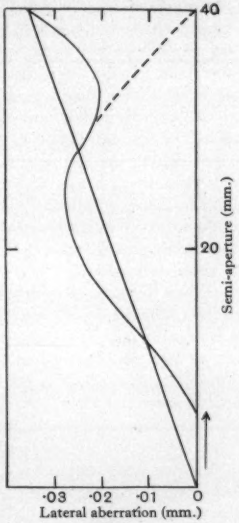


Fig. 4

This amount of aberration proved intolerable and in fact the objective was subsequently figured by removing glass from the outer zone; this actually increases the amount of undercorrection at the paraxial focus but at the same time enables a new focus to be found, ·88 mm. nearer the objective, where the greatest value of the residual aberration is $\lambda/15$. From certain other results described in the same paper it seems clear that in practice a residual aberration of $\lambda/4$ is a dangerously large amount to leave possible (during design) in the best focus of an optical system. The unavoidable irregularities in the system when made up may aggravate the phase errors and scatter a great deal of light outside the best focus. It is suggested by Prof. Conrady however that reduction of the residuals to amounts near $\lambda/6$ or $\lambda/8$ will be, in most cases, sufficient. When the wave is ideally spherical ·839 of the total light is found in the central disc, and when the phase residual is $\lambda/4$ mechanical integration shows that ·8 of this maximum possible intensity is still retained, or ·67 of the total energy.

In low amounts of aberration the phase of the resultant vibration at the best focus is near the mean of the extremes of phase from the surface. The intensity which results from two amplitudes A_1, A_2 with phase angle ϕ is

$$A_1^2 + A_2^2 + 2A_1A_2 \cos \phi.$$

Thus, when the extreme phase angle is down to about 60° or $\lambda/6$ further diminution of the phase differences will not greatly increase the intensity from the whole surface. This is the more apparent when we remember that the phase differences corresponding to other portions of the surface are all less than this amount.

Aberration on the Fresnel zone theory.

A discussion of the probable light distribution on the basis of the Fresnel zone theory is instructive but would occupy too much space to be given in detail. Although the results are not numerically accurate some predictions of interest can be made. For example it has been shown that with ordinary r^4 phase aberration the *margin* and centre of the wave are in the same phase at the 'best' focus, while the greatest variation is found in intermediate zones. As the margin is contributing actively to the best focus more or less independently of the amount of this r^4 aberration present it appears probable that the *size of the central spurious disc might be independent of the aberration*. This is verified in practice.

By a rough geometrical division of the wave into zones the persistence of light along the axis near the focus in the presence of aberration (in contradiction to the rapid diminution of the axial intensity to zero on each side of the focus with a spherical wave) is easy to show. This is due to the fact that when aberration is present there is generally (for the axial point considered) a fairly broad zone of uniform phase in the wave either at the centre (on one side of the focus) or at the margin (on the other side of the focus). The other zones are smaller and tend to annul each other.

When the zone of uniform phase is found around the centre, the distribution of light near the axis gives a diffuse hazy patch on one side of the focus, but when it is near the margin a well marked and bright diffraction ring system with a bright centre is found. In this way the general effects fit in well with our experiences of diffraction, although it is not well to lend undue weight to rough generalisations of this nature.

Fig. 5 will illustrate the rough geometrical construction to obtain the Fresnel zones for a wave (full line) having over corrected (r^4) spherical aberration in which the greatest phase residual is $\lambda/2$. Zones are found for a step inside the best visual focus sufficient to make the phase difference between centre and margin $dp = -\lambda$, also outside the focus to make $dp = +\lambda/2$ and $+\lambda$. The dotted lines represent circles with the best visual focus as centre; the phase difference between centre and margin vanishes at that point.

This short review of the subject may well be terminated by a reference to the work of Lommel which has been already mentioned. The complete series of curves for light distribution resulting from his integrations for the diffraction of a circular aperture without a lens yield an indication of a periodic variation of brightness in the ring system very similar to those obtained with a lens, except that the periodic extinction of light in the rings appears to be more complete in the case of the simple circular aperture than when the lens is present. The general reason for this similarity is not difficult to see, as the successive Fresnel zones can be shown to possess approximately equal areas whether the wave surface is plane or slightly spherical. The essential difference is of course that with a lens and the resulting curved wave surface the phenomena are symmetrical with respect to the centre of curvature. Lommel's diagrams, which were not seen by the present writer until the sketch

diagrams of light distribution near the focus of a lens had been drawn, certainly yield a great deal of information which can be applied to the case of a lens after consideration of the distribution of Fresnel zones.

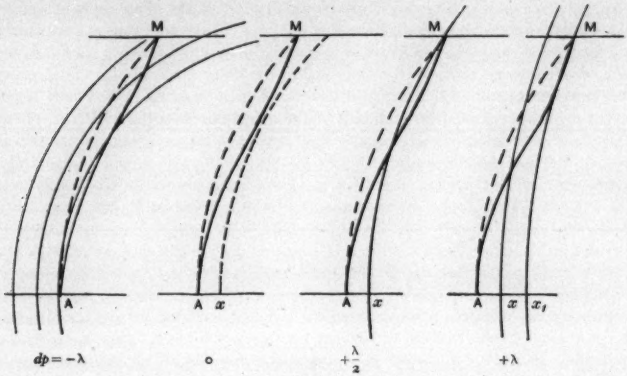


Fig. 5

PART II. EXPERIMENTAL STUDY

Performance of Instruments.

It is a matter of general experience that optical systems have often proved to be capable of a far better performance of resolving power than the calculated aberrations have been thought to indicate. The study of the imperfections of images must take account of the distribution of light as well as 'definition,' for it is well known that lenses differ greatly in the vigour and contrast produced in the image, a matter not wholly to be explained by multiple reflections or such unavoidable defects. The satisfactory explanation of these matters can only be given after the study of the images of sources which are of *such a small size that the relative distribution of light in the diffraction pattern does not appreciably change when the source is smaller.*

"Star" tests.

The so-called "star" tests on optical instruments, though widely known and used by opticians for testing the performances of telescope objectives, have received very little attention from physicists. Messrs T. Cooke & Sons (York) have issued a booklet* in which drawings of appearances of "star" images in and out of focus are given, including cases in which aberration is present. Michelson in a book on *Light waves and their uses* gives a few photographs of spurious discs and their

* *The Adjustment and Testing of Telescope Objectives.*

accompanying ring systems, the size of the system having been artificially enlarged by cutting down the aperture of the lens for producing the image. In view, however, of the great importance of the subject a scheme of work was undertaken at the Imperial College which aimed at the determination of the aberrations of one or two optical systems and the photographic recording of the appearances at and near the focus of a "star" image. Part of the work (already published*) related to the case of a telescope objective forming an image of an artificial star. That paper concentrated mainly on the fact (well known to opticians), that a similarity of the out of focus appearances on each side of the image is a valuable indication of the absence of aberration. The dissimilarity of these appearances when aberration is present is strikingly demonstrated by the photographs, and it should be noted that where aberration is present a great deal of the light apparently escapes entirely (on one side of the focus) from within the limits prescribed by the geometrical theory and approximately indicated by the outer bright ring. The appearances at the best focus were studied visually. It appeared desirable in any extension of the work to examine particularly the best focus appearances.

An alternative means of varying the amount of aberration in a system is to employ a microscope objective and use it at different working distances. Considerable amounts of spherical aberration may be obtained without a great modification of relative aperture, but on the other hand the difficulty of obtaining a suitable "star" object is much greater. A well-known old English method of testing microscope objectives employs the image of a small source of light reflected in very minute globules of mercury. This was tried but found difficult for any photographic work owing to the loss of light when the globule is sufficiently small. One may also employ one of the fine "pinholes" often found in a silvered or platinised piece of glass as an object on the microscope stage. It is necessary to exercise great care in selecting the pinhole to be employed. It must be in a sufficiently opaque part of the film, so that the out of focus appearances may be seen with sufficient contrast; its size must be so small that its "perfect" geometrical image would be small compared with the dimensions of the spurious disc; it must distribute the light very uniformly into the objective (*most* important) and yet it must pass sufficient light for photography with exposures of reasonable length.

It is necessary to select the pinhole very carefully with the objective and a high power eyepiece, the silvered glass being used on the stage, with the adjustment of illumination as perfect as possible. For visual observation the illumination need not be intense and an ordinary frosted electric lamp will serve, but for photography it was found best to illuminate the pinhole by forming upon it an image of the tungsten ball of a Pointolite lamp, by means of the microscope condenser with an auxiliary collimator. For projection of the image several so-called "projection eyepieces" were at first tried but had to be rejected as they were all astigmatic. However, as the light entering the optical system was confined in the present case to that from a few pinholes there was little objection to using an ordinary Huygenian

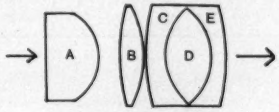
* Mrs C. H. Griffiths, "Diffraction Patterns in the Presence of Spherical Aberration," *Trans. Opt. Soc.* 21 (1919–20) 87.

eyepiece, and an excellent 18 mm. lens of this type was used for most of the photographs. The focussing was controlled by a Ramsden eyepiece held in position by a piece of plain glass used instead of a focussing screen. A special Wratten colour filter (green) was employed.

Having established the possibility of obtaining photographic records by such means, it remained to select and determine the aberrations of a suitable lens.

Determination of the aberration of a Microscope Objective and the variation of Aberration with working distance or "tube length."

Through the kindness of Prof. Conrady a microscope objective which had been constructed to an available specification was secured. This is shown (enlarged) in the diagram (Fig. 6). It consists of two front-lenses with a triple back lens. The focal length is 8 mm. and with its relatively large *N. A.* (·65) and back aperture (1 cm.) it is comparatively sensitive to variations of tube length, although the eight surfaces render possible a good spherical correction.



A, Barium Crown Glass
B and D, Crown Glass
C and E, Flint Glass

Fig. 6

The aberrations could have been calculated entirely from the constructional data, but this would have allowed nothing for errors in construction. It was therefore decided to calculate the axial and phase aberrations for a marginal ray for a few tube lengths, and to try a modification of the Hartmann test in order to check the computations and determine the aberrations for the whole of the aperture. The ray tracing was performed (by the usual equations with suitable checks) for working distances of 6, 6·9 and 8·5 inches from the back lens, and the path difference (phase difference between the marginal and paraxial path disturbances at the marginal focus) was calculated by means of the formula given by Prof. Conrady*. The following results were obtained:

Table I.

Working distance	$1/L$	Phase difference (paraxial path—marginal path)
6·0 inches	·166	− 4·11λ
6·9 „	·145	− 0·4 λ
8·5 „	·1175	+ 4·4 λ

Plotting $1/L$ against phase difference (Fig. 7) it is seen that the relation is very nearly linear. This important property (which holds generally for microscope objectives) does not seem to have been previously recorded, although Prof. Conrady informs me that he was familiar with the existence of the relation from past experience.

In making the above calculations the utmost care was taken to ensure accuracy. The results for each surface were verified by independent checks, and the results

* Equation (8) part 1 (p. 67).

for the phase differences arising at each surface were also checked by separate formulae.

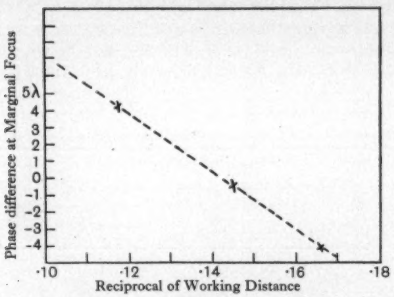


Fig. 7

In applying the above results it would be pedantic to expect to find zero aberration, indicated by complete symmetry of extra focal appearances, at the exact point indicated by the above work. It is, however, legitimate to ascertain the minimum aberration point experimentally (it fell close to the theoretical position) and to assume that the variations of phase differences will follow the computed law (fairly nearly) with regard to displacements from that point. With an objective of fairly large numerical aperture the best tube length, indicated by the closest approach to symmetry of the out of focus appearances in a short range on each side of the focus, is not hard to ascertain within one or two millimetres when using such an artificial star test object and when the zonal aberrations are not prominent: otherwise considerable difficulty may be experienced. Having found the marginal ray phase differences at the marginal focus the residuals at the best focus, assuming the aberration introduced by change of tube length to follow the fourth power law, may at once be calculated, as already shown in Part I of this paper.

Table II.

True tube* length	10/*L*	Tube* scale	Calculated residual at best focus ($\frac{1}{4}$ marginal *W*) for full aperture	Final† estimate of effective phase residuals. (Short exposure photographs)
18·0	·556	14·26	− 0·68λ	−0·32λ
19·64	·508	15·9	− 0·03λ	−0·01λ
19·74	·507	16·0	0·0λ	0·0λ
20·24	·494	16·5	+ 0·17λ	+0·08λ
20·98	·477	17·24	0·42λ	0·20λ
21·81	·459	18·07	0·66λ	0·31λ
22·74	·440	19·0	0·92λ	0·44λ
23·24	·430	19·5	1·05λ	0·50λ
24·08	·416	20·34	1·2λ	0·58λ

* The ordinary eyepiece holders were not used.

† The derivation of this column is yet to be explained. It is to be noted that the figures in the second place of decimals are uncertain except in their *relative* significance.

This table was obtained on the assumption that the best tube length lies at 16·0 but the possibility of fixing this position to within a few millimetres in this way depends greatly on the shape of the wave curve produced by the objective, as was pointed out above. If residual zonal aberration *is* present, the extrafocal appearances will nowhere be completely similar and a doubtful region of tube length is found, somewhere near the centre of which the position of zero marginal aberration must be assumed to lie. That *slight* zonal aberration was present in this case was evident from a careful examination of the image.

Further these figures are based on ray paths calculated for the extreme edge of the lens. In practice these edges are especially liable to become rather too rounded in polishing and we shall be doubtful whether such an effect as this has not affected the wave form. Further also the light passing through the outer zones is constantly meeting glass surfaces at greater angles of incidence than that nearer the axis and consequently it tends to become weakened. We are therefore at present uncertain how far the particular appearances due to zonal aberration and the main aberration in the margin of the wave have predominated in fixing the best focus. Evidently little can be done unless a practical test on the lens is carried out, but by reducing the total aperture to 9 mm. the indication of 16·0 as the best tube scale reading for minimum aberration was much more definite.

Adaptation of the Hartmann test.

To carry out such a practical test a somewhat larger "pinhole" was selected for an object and a small diaphragm was constructed to fit into the objective mount near the back lens. This diaphragm had drilled in it on three diameters small holes, pairs of which were the following distances from the centre:

1st diameter	1, 3, 5	mm.
2nd	,, 2, 4	mm.
3rd	,, 2·5, 4·5	mm.

The diameter of each hole was just under 1 mm. The whole diaphragm was made for me with very great care by Mr Perkins. The holes had edges bevelled on one side and the metal was finished dead black. The ordinary eyepiece of the microscope was removed and a small plate holder with a draw screen was obtained which would fit into the same tube. The rationale of the Hartmann test* is of course to expose a plate on each side of the focus with a diaphragm (such as the above) in position. Diffraction images are formed on the plate corresponding to each aperture, and the centres of such images may be taken as marking the ray paths corresponding to the centres of the holes. If l_1, l_2 be the distances between the images corresponding to the same pair of holes on each side of the focus then the distance f from the first position of the plate to the focus for the zone in question for the pair of holes is given by

$$f = \frac{l_1}{l_1 + l_2} \cdot d.$$

The displacement of the plate holder was secured in the present experiments

* *Astrophys. Journ.* 25, p. 195.

simply by the draw tube. The possibility of doing this is very dependent on the stability of the relative positions of objective and object while the rack of the draw tube is worked. This point was carefully tested and the focus was found not to change appreciably. This would not be the case with every microscope, unfortunately, but in cases of difficulty the sensitive plate could be supported and moved quite independently. Ordinary rapid plates with a yellow screen were employed and after a certain number of preliminary attempts it was found possible to obtain measureable plates. A very considerable displacement was necessary in order to get sufficient separation of the diffraction images to prevent mutual interference effects. It was decided to determine the aberrations for two working distances, one for nearly minimum aberration as revealed by visual examination and the other for a considerable amount of over-correction (estimated from the table given above to have a value of about $4{\cdot}0\lambda$ *at the marginal focus*).

The "effective" wave length is in the blue green region, thus results cannot be expected to accord *strictly* with those obtained by methods involving the use of other wave lengths. The only reason for not employing monochromatic light is the somewhat prohibitive length of the necessary exposure.

In practically every microscope objective there are small centering errors and small amounts of cylindricity in the surfaces which produce more or less axial astigmatism. These amount to very little in terms of phase difference; their only effect is a slight distortion of the spurious disc pattern, but in order to allow for these it is best to determine the aberrations independently for two mutually perpendicular diameters of the lens for each pair of apertures, which necessitates the securing of four plates for each focus. This was done in the present work.

In securing the photographs the 18 mm. eyepiece was first of all placed in position at the correct tube length (the diaphragm being in place) and the focus obtained by the main rack and fine adjustment. There is no difficulty in this. The plate holder was then substituted and the tube racked in or out until the diffraction images in the plane of the plate observed by a magnifier were well separated, the exact position of the tube being noted. Slight adjustments of the condenser to control the distribution of the light in the diffraction images were occasionally necessary. The resulting photographs were carefully measured with a photo-measuring micrometer. To and fro readings were made in measuring, two edge settings and one centre setting being made each time on the central discs of the diffraction images.

The necessity of using an ordinary microscope for the work was unfortunate as in substituting the plate holder for the eyepiece and using the tube rack the position of the focus is inevitably liable to small disturbances. Though these were found to be small, they prevent the possibility of correlating the positions of visual and paraxial foci. There was also inevitable disturbance in changing the orientation of the diaphragm, as the objective had to be removed and the whole of the adjustments repeated.

Table III.

Tabular results of Hartmann tests on 8 mm. objective. The 2 mm. zone focus is taken to be the paraxial focus.

1st focus (adjusted by 18 mm. eyepiece). Tube reading = 16·5. (Near minimum aberration.) Displacement of plate = 19·15 cm. Exposure = 15 mins. Distance from diaphragm to paraxial focus in 1st position of diaphragm = 19·82 cm.; in 2nd position = 20·02 cm. Mean distance from diaphragm to paraxial focus = 19·9 cm.

Aberration Table (*Measurements with respect to paraxial focus*).

Zone	1st position of diaphragm. Axial	2nd position of diaphragm. Axial	Mean axial aberration	Lateral aberration. Mean value in cm.
1·0	0·0 cm.	0·0 cm.	0·0 cm.	0·0
2·0	− 0·063	+ 0·032	− 0·015	− 0·00015
2·5	+ 0·001	+ 0·037	+ 0·019	+ 0·00024
3·0	− 0·030	+ 0·121	+ 0·045	+ 0·00068
4·0	+ 0·064	+ 0·131	+ 0·097	+ 0·0019
4·5	− 0·071	+ 0·006	− 0·032	− 0·00072
5·0	(Images not well enough defined to measure)	− 0·184	− 0·184?	− 0·0046?

2nd focus (adjusted by 18 mm. eyepiece). Tube reading = 19·5. Estimated marginal aberration $W = 4\lambda$ at marginal focus. Total displacement of plate = 13·95 cm. Exposure = 20 mins. Distance from diaphragm to paraxial focus in 1st position of diaphragm = 22·0 cm.; in 2nd position = 22·4 cm. Mean distance from diaphragm to paraxial focus = 22·2 cm.

Aberration Table.

Zone	1st position of diaphragm. Axial	2nd position of diaphragm. Axial	Mean axial aberration	Lateral aberration. Mean value in cm.
1·0	+ 0·0 cm.	+ 0·0 cm.	+ 0·0 cm.	+ 0·0
2·0	+ 0·060	+ 0·199	+ 0·129	+ 0·0012
2·5	0·177	0·192	0·184	0·0021
3·0	0·277	0·381	0·329	0·00445
4·0	0·565	0·678	0·621	0·0112
4·5	0·714	0·808	0·761	0·0154
5·0	0·770	0·764	0·767	0·0172

This method of carrying out the Hartmann test for a microscope objective is believed to be new.

It may be remarked that the exact position of the focus could hardly be repeated for both positions of the diaphragm, but the run of the aberrations must be very similar for small ranges of tube length and there is nothing to prevent taking a mean value of these.

Referring to the table and curves for the two cases it will be seen that the aberrations in the first case are generally small and in the second case comparatively large, at least with respect to the paraxial focus. The regularity of the results in the two cases is encouraging. The astigmatic discrepancies between the aberrations for the two positions of the diaphragm are not greater, considered in terms of phase differences, than the indications in the image seem to justify. We have, however,

to explain the occurrence of best visual focus for the wave form *A* (Fig. 8), and also to examine the indications of the Hartmann test as to the amount of aberration in the second case, and compare this with the results of computation.

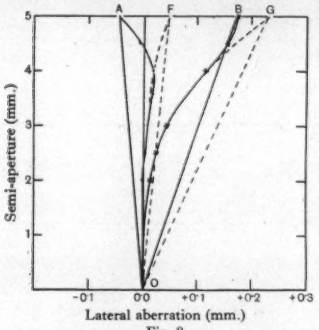


Fig. 8

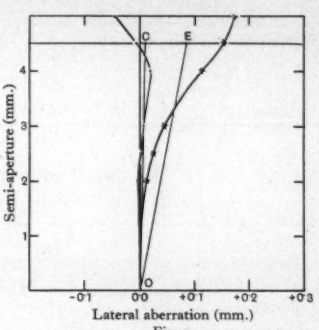


Fig. 9

Plotting lateral aberrations against aperture (Figs. 8 and 9), we can easily evaluate phase differences with the aid of a planimeter, on the principles already described. Drawing lines *OA*, *OB* to represent marginal foci, we find from the relative areas of the curves phase differences of $-1{\cdot}3\lambda$ and $+1{\cdot}7\lambda$ respectively, which is apparently not at all in agreement with the results of visual observation. An important fact, however, was clearly shown by the photographs of the diffraction images, namely, that the intensity of the light from the last half millimetre of the aperture is very faint indeed.

This zone, then, cannot play an important part in fixing the position of best focus. If we imagine that it is missing and the aperture limited to 9 mm. the greatest phase residuals for foci *OC* and *OE* (Fig. 9) are reduced to $\cdot1\lambda$ and $\cdot4\lambda$ approximately, which would indicate that in these circumstances the minimum aberration will be found when the adjustment is not far from that which gives the wave form *OA* (Fig. 8). If, however, the best focus *is* here and the indications of the curve are correct, the light from the outer zone must be quite considerably out of focus, and it should be possible to test this point experimentally by comparing the focussed images at full aperture and when the aperture has been reduced to 9 mm. This was actually done and it was found that practically the only function of the last half millimetre zone is to scatter light into the outer diffraction rings at the best focus. The image is very greatly improved by slightly reducing the aperture. A series of photographs with long exposures and differing tube lengths has been taken to illustrate this effect, but the effect would probably be lost in reproduction. The proof of the point is, however, quite definite.

As regards the marginal phase differences, their correspondence with the theoretically computed values will evidently depend entirely on the complete

absence of any increased curvature towards the edge of the lenses. For example, if we disregard the results for the 4·5 and 5·0 mm. zones, producing the curves naturally to *F*, *G* (Fig. 8) the amounts of marginal aberration in this case become $+0{\cdot}8\lambda$ and $+3\lambda$ as against $0{\cdot}7\lambda$ and 4λ expected from the computation*: Without labouring the speculation unduly, sufficient has been said to show that if the bends in the curve are chiefly due to a certain amount of edge error in the surfaces the results are fairly well in accord with theory. Slight discrepancies in the setting of the lenses to the prescribed relative distances may well account for any remaining differences.

We can now draw a sketch to show (diagrammatically) the nature of the departure of the wave from the spherical form. Fig. 1 (as a matter of fact) illustrates the conditions for long tube length. We now have over-correction and positive lateral aberration while at the same time the wave has a lead at the margin with respect to the marginal focus indicating a positive "*W*" value (see equation (8) of Part I).

From the results of the Hartmann test we can now make an estimate as to the actual effective amounts of residual aberration present in the conditions under which the photographs were made. For comparatively short exposures or for reduced aperture we may, for the reasons explained, neglect the contributions of the outer $\frac{1}{2}$ mm. zone and take the effective phase residuals as those in the remainder of the aperture. Computing the effective values of the residuals up to 9 mm. aperture as $0{\cdot}5\lambda$ at the focus indicated by *OE* (Fig. 9), corresponding to the tube scale 19·5, we can calculate a final series of effective residuals which is quoted in Table II, and which will certainly not give values which are too low.

It will now be desirable to proceed to a discussion of the characteristics of the spurious disc images under different conditions of aberration. In all cases where photographs have been taken the focus was adjusted till the brightness of the central disc as compared with the surrounding rings appeared to be a maximum.

Dimensions of Diffraction Pattern.

A series of photographs (Fig. 10, B) was made with varying amounts of spherical aberration present, the focus being adjusted visually by examining the image in the plane of the plate with a magnifier. Care was taken to ensure that the image of the diffraction pattern should be always photographed under the same magnification. The numerical value of the magnification was found by the use of a stage micrometer with identical conditions of tube length, etc. The "first" magnification, produced by the objective alone, was found to be 23·7 in the conditions at the time (this varies with the tube length). The total magnification was 355, indicating a necessary magnification by the projecting ocular of 14·96, or 15 sufficiently nearly. (The reproduced photographs are all photomicrographs of the images on the plates, the total magnification being many thousands.)

The dimensions of the photographed diffraction patterns on the plates were

* The marginal "*W*" from Table II corresponding to tube scale readings of 16·5 and 19·5 respectively.

found by means of a micrometer microscope, measurements being made in two directions at right angles in order to get a mean result which is less affected by the presence of a little irregularity in the patterns (doubtless the effect of an error in centering of one of the lenses of the objective). This irregularity was quite independent of any ocular which was employed and also was the same with different pinholes. In Table IV the experimental and calculated values of the diameter of the first dark ring are given. The calculated values were of course obtained from the formula

$$D = \frac{2{\cdot}44\lambda f}{a}$$

where λ = wave length (assumed = $\cdot 5\mu$).

f = distance from aperture to image.

a = diameter of aperture.

The photographs were taken with the 9 mm. diaphragm in position. The measured diameters of the projected images on the plates were of course divided by 15 to obtain the true value of the rings themselves.

Table IV.

Tube length	Residual Aberration present	Observed Diameters 1st bright ring	1st dark ring	Calculated Diameter 1st dark ring	Remarks
	λ	mm.	mm.	mm.	
18·0	− 0·32	·036	·027	·0267	1st bright ring fairly intense
19·64	0·10	·038	·028	·0288	,, feeble
20·24	< 0·10	·041	·030	·031	,, feeble
20·98	+ 0·20	·042	·032	·0318	,, somewhat stronger
22·74	0·44	·051	·037	·034	,, very feeble
24·74	0·75	·052	·038	·037	,, fairly strong

The value of λ from which the diameters should be calculated is of course a little uncertain. The photographs (see Fig. 10, A) were made with a Wratten yellow filter No. 9 and Ilford panchromatic plates, and the position of the dominant wave length is near $\cdot 5\mu$; any uncertainty would quite easily explain the small disagreement between observed and calculated values. For the fifth photograph the spurious disc is larger than it should be according to the general run of the results.

We observe that the size of the first dark ring at the best visual focus is not greatly affected by the presence of residual aberration amounting to $0{\cdot}75\lambda$.

KEY TO FIGURE 10.

A. Focus discs

	1	2	3	4	5	6
Tube length	18·0	19·6	20·2	21·0	22.7	24·74
Residual aberration	−0.33	−0·1	0·1	0·19	0·43	$0{\cdot}75\lambda$

B. Extra focal appearances. (Aberration a minimum.)

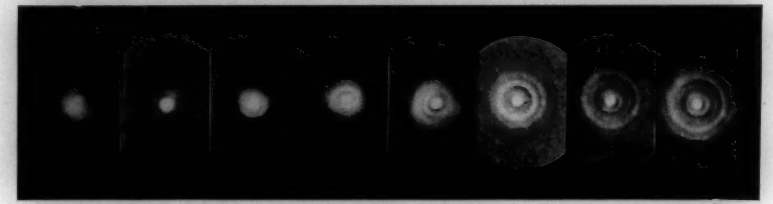
	1	2	3	4	5	6	7	8	9	10
$\delta\rho =$	−2·6	−1·6	−1·0	−0·7	0·0	+0·25	+0·7	+1·0	+1·4	$+2{\cdot}1\lambda$

C. Extra focal appearances. (Aberration residual at best focus = $\lambda/2$.)

	1	2	3	4	5	6	7	8
$\delta\rho =$	−1·0	0·0	0·5	1·0	1·45	1·8	2·25	$2{\cdot}5\lambda$

A 1 2 3 4 5 6

B 1 2 3 4 5 6 7 8 9 10



C 1 2 3 4 5 6 7 8

Fig. 10

Taking the results of the Hartmann test and finding the diameters of the geometrical circle of confusion for various positions of the focal plane between the paraxial and marginal foci (which may obviously be found very simply from the curve by taking 'x' intercepts between the curve and the line representing a particular focus) we find that the circle of confusion half way between paraxial and marginal foci is about 0·15 mm. when the tube length is about 23 cm. (This is considering the 4·5 mm. zone as the margin.) That 'half way position' is the point at which the confusion of *phase* is nearly a minimum, which is a strong a priori indication of the position of the best visual focus. The circle of *least* confusion proves to have a diameter of ·09 mm. and is found rather nearer the marginal focus, but both these circles are large compared with the observed diameter of the spurious disc which is ·037 mm. measured to the middle of the dark ring. It is therefore perfectly useless to calculate the loss of light from the disc image in terms of geometrical ray paths.

A careful examination of the photographs suggests, however, that in the presence of aberration the central disc at the best focus appears to be smaller in comparison with the dark ring, as though the intensity increased much more rapidly in going from the dark ring towards the centre when aberration is absent. In the case of the fifth photograph an effect which is quite familiar has come into play. On going outside the focus in one direction when aberration is present the central disc increases in size and the surrounding ring loses intensity, being merged in a general diffuse haze round the centre concentration. On going a little further the central disc itself disperses with great rapidity. The focus in No. 5 (Fig. 10, A) was evidently imperfectly adjusted and the surrounding light ring was lost in the haze, which was not reproduced in the photograph. It may, however, be taken as proved that even for considerable amounts of aberration the eye will generally choose the position of best focus where the size of the diffraction pattern corresponds to that given by the usual formula. The physical effect of aberration at this "best focus" is simply to scatter more light away from the central disc. This conclusion confirms Prof. Conrady's numerical results. The persistence of the central spurious disc explains the nature of the fairly defined images surrounded by "flare" so characteristic of lenses possessing aberration.

Distribution of Light in the Diffraction Pattern.

Mrs Griffiths* has described a convenient comparator apparatus for obtaining an estimation of the intensities of different parts of ring diffraction patterns. A microscope has an arrangement by which the images of parts of two photographic plates may be projected, each into one half of the field of view.

For comparing intensities of disc and ring in the present work it was decided to use the photographic comparison method only. For comparison plates, contact prints of a previously calibrated neutral annulus (described elsewhere†) were made with such an intensity of light that the exposure and treatment of the plates could be in all respects identical with that of the plates on which the star images were

* *loc. cit.* † Abney, *Researches in Colour Vision*, p. 70.

photographed. The annulus gives a range of intensity from 1 to ·02 or (50 to 1) in its circumference of 360°. One print was therefore made at 1 ft. from the source of light and a second at a distance of $\sqrt{50}$ feet, which obviously gives a continuing range of ·02 to ·0004. The exposures were adjusted so that the ·0004 position just affects the plate. In making the comparison a fair magnification is necessary owing to the extreme smallness of the diffraction images. The grain of the plate is obtrusive and the comparison is very difficult to make satisfactorily. More reliance has to be placed on the closeness of packing of the silver grains than the photometric intensity or opacity of the image.

It must be noted that in the present case the size of the spurious disc changes with varying tube length. It is not permissible, therefore, directly to compare the intensities of the discs as the intensity of light in the discs varies with the tube length and also, obviously, with the working numerical aperture which is not quite constant, although the changes owing to the latter cause are shown to be small from the results of the calculation.

The small size of the images unfortunately makes even the determination of the maximum intensity of the first ring relative to that of the disc a matter of some uncertainty. A kind of average (not maximum) brightness is all that can be estimated.

Results of Photographic Estimation of Intensities.

Sets (*a*) and (*b*) (Table V) were entirely independent experiments, the density comparison plates being made independently. Different developers were used. Pyro-soda was the developer for all plates in set (*a*), metol-hydroquinone for all plates in set (*b*). The exposure times for the star images and comparison plates were different in the two sets, denser star images being obtained in set (*a*). Tank development was used throughout. The relative intensities which would result merely from the change of size of the disc if *all* the light were present is listed in the third column and the ratio "1st ring brightness/central disc brightness" in the fourth column of set (*a*).

Table V.

Set (*a*)				Set (*b*)		
Residual aberration	Central disc	Calculation	Ratio ring/disc	Residual aberration	Centre disc intensity	1st ring intensity
− 0·32λ	0·5	1·2	·007	− 0·32λ	< 1	< ·009
− 0·1λ	1·0	1·0	< ·004	0·1λ	1	< ·009
+ 0·08λ	1·0	1·0	< ·004	0·08λ	1	< ·006
0·20λ	1·0	0·88	< ·004	0·20λ	< 1	< ·006
0·44λ	0·8	0·75	·001*	0·3λ	< 1	< ·009
0·75λ	0·3	0·63	·007	0·6λ	< 1	< ·018*

The photographs from which all these results were obtained were taken after the focus had been adjusted visually, so that the relative intensity of the central disc appeared to be a maximum. The two results marked with an asterisk are apparently low and high respectively owing to a small inaccuracy in judging the focus which produces a great change in relative amount of light in the ring.

The broad indications are, then,

(1) Energy disappears from the central disc at the best focus when spherical aberration is introduced.

(2) The actual intensity of the first bright ring (always at the best visual focus) appears not to be greatly increased when aberration is introduced. The scattered light from the central disc is not therefore found in the rings, but is distributed further away into the field.

The difficulty of obtaining satisfactory numerical results for the central disc intensities by photographic methods will perhaps be doubted in view of the familiarity of a common astronomical measurement, i.e. "stellar magnitude." In that case, however, the total intensity only is required. A long exposure is given and the spreading of the darkened portion of the image due to halation, light in the diffraction rings, etc. gives a measure of the stellar magnitude in relation to other known stars appearing on the same plate. Where differentiation is required, however, between light actually in the central disc and that in the rings or in haze surrounding the central concentration, the method of long exposure is not valid.

Measurements by "Extinction" Methods.

The extinction of light method (similar to that employed for measuring stellar magnitudes) at first sight offers a method of comparing the intensities of the central discs in various conditions of aberration by employing the "threshold of vision" as the source of a kind of natural light unit, especially since the eye can recognise easily the haze, diffraction rings, and central disc if a sufficient magnification is employed. It is interesting, however, to consider whether the intensity of the central disc alone can be measured in this way. The angle subtended at the eye by the diameter of the first dark ring when a 7 mm. eyepiece is employed is obviously ·03/7 for the case of small aberration, or 13′ 51″. We may use an apparatus to control the intensity of the light, diminishing it until the star disc becomes invisible, the haze and rings, and even the round shape of the disc, being indistinguishable near the threshold.

The experiments of Paterson and Dudding* have established that the visibility (the reciprocal of the threshold) of point sources subtending angles under 1 minute is proportional to the total amount of light entering the eye from the source. Moreover the presence of a slightly illuminated background means only a fairly uniform 10 per cent. decrease in visibility for a range of brightness of background from ·004 to ·02 metre candles.

Experiments by myself† on a similar problem have indicated that the additive effect of the light in determining the visibility of small sources may extend over an angular area subtending as much as one degree, but this result applied to cases in which the distribution of illumination throughout the area was uniform.

The following table gives results of extinction values for the best focus concentration. They must not be interpreted as necessarily applying to the central disc

* *Phys. Soc. Proc.* vol. 24, Oct. 1912.

† L. C. Martin, "A Study on the Performance of Night Glasses." Dept. of Scientific and Industrial Research, *Bulletin*, no. 3.

alone, but to the light in a limited area outside. Experiments were begun after about twenty minutes in a darkened room and were repeated in a "reversed" direction. The intensity of light passing into the microscope was controlled by an annulus.

In taking extinction results the use of the same part of the retina was constantly secured by foveal fixation on a definite mark in the field of view about 2° from the star.

Table VI.

Amounts of light in a region surrounding the best visual focus, with and without aberration.

	Series (1). Eyepiece 7 mm. Angular subtense of spurious disc = 14′ for zero aberration tube length				Series (2). Eyepiece 18 mm. Subtense of spurious disc = 5′.4	
Tube length	Residual aberration	Mean* annulus readings 'Forward'	'Backward'	Indicated intensity†	Residual aberration	Indicated intensity†
18·75	− 0·2λ	59°	79°	0·97	− 0·15λ	0·95
20	< 0·1λ	63°	65°	1·0	+ 0·08λ	1·0
21	+ 0·20λ	89°	76°	0·87	+ 0·37λ	0·64
22	+ 0·3λ	101°	88°	0·77	+ 0·6λ	0·59
23	+ 0·45λ	111°	118°	0·60	+ 0·65λ	0·49
24	+ 0·6λ	120°	120°	0·58		

The foregoing results considered together, then, appear to indicate that for an aberration at the best visual focus of about quarter of a wave length at least 20 per cent. of the light disappears from the central condensation and is scattered far‡ into the surrounding field, and when the aberration increases to about half a wave length some half of the light is lost in this way. In neither case, however, would the telescopic resolving power (depending only on the size of the spurious disc) be diminished, and a telescope object glass afflicted with aberration of such amounts might pass as a good one if the scattered light is sufficiently widely spread as to be of lower intensity than is appreciable to the eye. In the use of any lens, however, with aberration of even very small amounts the loss of contrast in images will reveal the defect very quickly, especially if there is a great deal of light in the field of view. The lunar landscape should make a magnificent test object for a telescope objective. I have been informed that it is a matter of practical experience that some object glasses which perform well on double stars are almost useless for planetary detail.

It appears, therefore, that for the securing of best results from a lens system it must be so constructed that the residual aberrations at the best focus are brought within a sixth or eighth of a wave length, this best focus being, of course, not necessarily the paraxial or marginal focus. This conclusion has been previously reached on theoretical grounds; it may now be regarded as definitely established.

* Each is the mean of six results. The probable error of the final mean is of the order of ± 5°.

† The intensity in both series is referred to 1·0 as the maximum in the table and has no reference to the total light.

‡ That is to say, the light is not found in the interference rings immediately surrounding the central condensation, being spread over a much greater area. With bright "stars" it appears as a nebulous haze.

Extra-focal Effects.

While the effects at the best visual focus in the presence of aberration are probably of greater practical importance, the appearances at short distances on each side of the focus are of extreme interest. The series of photographs given by Mrs Griffiths shows the broader effects, but the exact tracing of the changes of light distribution is desirable.

The method adopted for investigation was to use the 7 mm. eyepiece which magnifies the diffraction pattern sufficiently for the various changes to be traced. The tube length was first adjusted so that the aberration is a minimum, and the focus could then be altered by the tube rack. It is convenient to describe the changes in passing from the position of best focus in one direction as the changes are observed to be practically identical on each side of the focus when the aberration is a minimum.

The distance from the focus is best described in terms of the optical difference of path between a marginal and paraxial ray which would be involved in such a displacement. This enables the change in the distribution of the half period zone to be followed. The above difference of optical paths is given by

$$dp = \frac{1}{8}\, df \left(\frac{A}{f}\right)^2 .$$

For this work a Wratten P filter was used. Dominant wave length transmitted $= \cdot 52\lambda$ (a narrow range only; the light may be considered monochromatic). Photographs of the effect are given in Fig. 10, B and C.

In examining the out of focus effects with a low power eyepiece the small ring structure surrounding the central concentration appears to be lost in a disc of light of which the diameter increases proportionally to the distance from the focus. If aberration is present the disc may show greater illumination at the edge, centre, or in intermediate zones, and a system of broad circular diffraction bands appears to emerge from the centre as the disc expands. These dark diffraction bands are much more marked on one particular side of the focus. If the aberration is small, the phenomena are hardly so striking, but are present on each side of the focus. Note that the total phase difference should best be calculated in successive steps from the best focus of not more than 1 mm. The formula otherwise gives slightly incorrect results.

It is of great interest to examine the appearances near the focus under greater magnification. As is naturally expected, the illumination of every point seems to undergo a periodic variation in intensity as the reference plane passes steadily through the focus and outwards. An interesting feature is that the different bright rings found in the Airy pattern appear to maintain their individuality while suffering periodic changes of brightness and size. Light appears in rings further and further from the centre, the further one goes from the best focus.

The first sketch diagram (Fig. 11) shows a visual estimation of the relative alterations in brightness at points in the diffraction pattern on going inside and outside the focus of the same objective when the aberration is a minimum. Up to

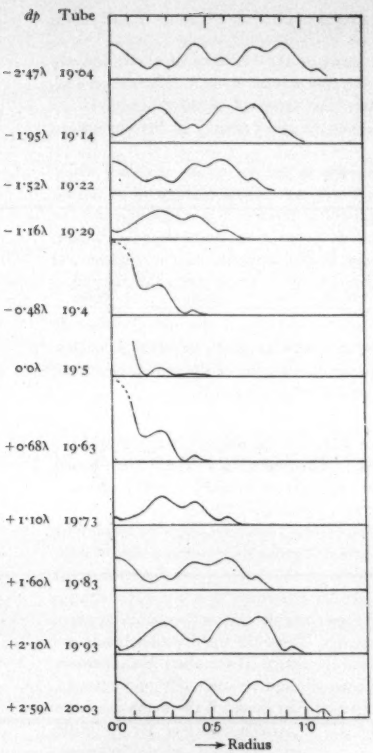


Fig. 11. Distribution of Light near the Focus. Aberration a minimum for λ = ·52.

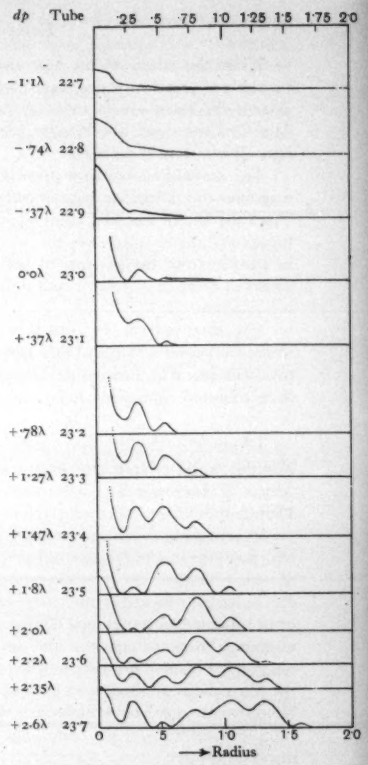


Fig. 12. Distribution of Light near the Focus. Phase residual at best visual focus = λ/2 for λ = ·52.

In both Figs. 11 and 12 the ordinates of the curves 'away from the best focus' have been exaggerated to show the distribution of the light in the particular diffraction patterns more clearly. They are largely the results of visual estimation. In reality the volumes of the spaces formed by rotating the figures round the central line should be constant.

$dp = 0{\cdot}7\lambda$ the first and second rings appear to absorb light at the expense of the central disc. At $dp = 1{\cdot}1\lambda$ the central disc appears of minimum intensity, or a hardly visible pinpoint of light at the centre of a dark ring. At $1{\cdot}6\lambda$ the centre has again grown while the first bright ring has almost vanished in the centre of the

first of the broad dark rings which grow out from the centre. The first bright ring now grows again while the centre and second ring shrink, only to grow again, thus making the broad rings appear to travel outwards from the centre. Owing to these variations in intensity and size of the bright elementary rings the dark rings appear to swing to and fro while varying in intensity. The spreading of the broad diffraction rings is thus due to an action amongst the elementary dark rings very much resembling the action of the elements of a harmonic wave machine for illustrating the propagation of longitudinal wave motion.

When spherical aberration is introduced (Fig. 12) the changes found outside and inside the focus are very dissimilar. Over-corrected spherical aberration is introduced by increasing the tube length. *At* the best focus the only change due to aberration that can be detected when a very bright star is employed is that a broad but perceptible haze of light surrounds the focussed image. This is not registered in the photographs owing to its relatively low intensity. Outside the focus we find that the central bright spot persists, and though it appears greatly to diminish in size it is brighter than the remainder of the diffraction pattern up to a path difference $dp = 3{\cdot}0\lambda$ (approx.), when, as in the present case, the spherical aberration present appears to mean a residual phase difference at the best focus of about $\cdot 5\lambda$.

The variations are slower then than in the case of the best tube length and zero aberration, but the outward progression of the dark rings proceeds in a similar and, in fact, a more striking way, as the variations in intensity in the diffraction pattern are much more marked when aberration is present.

Inside the focus, as we should expect from the approximate theory, there is also a persistence of light in the centre, but the sharply definite ring system disappears and a mere nebulous haze takes its place. The light appears to be thoroughly disorganised*, if such a term might be employed. The diagram (Fig. 12) will give an idea of the relative changes in the distribution of the illumination.

Results of Experimental Study.

Investigations, both numerical and experimental, of the monochromatic aberrations of an 8 mm. microscope objective are described, including a new application of the Hartmann method. The results are employed to check the computed values of phase residuals found at the focus for various working conditions. At a slightly reduced aperture the phase variation may be considered as proportional to the fourth power of the aperture.

The appearances at the visual focus with varying amounts of aberration were examined both visually and from photographs. The dimensions of the first dark ring are shown not to vary perceptibly from the normal even with a phase residual of $\cdot 75\lambda$†. The maximum resolving power would therefore still be about the same

* That is, the local irregularities in the wave surface from the experimental lens now produce a greater relative effect. There is no longer the regularity necessary to produce a symmetrical light distribution.

† An unexpected result from the geometrical point of view! In the booklet on *The Adjustment and Testing of Telescope Objectives* (Cooke, York) it is stated that:—"doubtless the effect" (of spherical aberration) "would be to make the disc considerably larger."

(but as this depends partly on the distribution of light in the central disc there would probably be some slight modification).

The diameter of the spurious disc is also very much smaller in the present case than the diameter of the circle of least confusion.

Examination of the photographs and further visual experiments show that when aberration is introduced energy disappears from the central disc, and at the best visual focus is apparently spread out considerably in the field, as the actual intensity of the first bright ring is not greatly, if at all, increased and the dark rings still appear dark. The results of the work point again to the necessity of considering the phase residuals as the only satisfactory method of expressing optical aberrations. In order to secure the placing of the full amount of light into its correct place in the image such phase residuals should be brought within $\frac{1}{4}\lambda$ at the best visual focus. While the "Rayleigh limit," or even a far worse degree of aberration, does not necessarily interfere with the resolving power of a lens system (on the axis, for monochromatic light) yet at the "Rayleigh limit" when the residuals at the *best focus* amount to $\frac{1}{4}\lambda$ it is now proved numerically and practically that 20 per cent. of the light has disappeared from the central disc and is distributed in the field where it forms a haze, weakens contrasts, and makes the image tend to be increasingly "watery" and unsatisfactory. It is true that the diminution of the intensity of the centre disc was recognised by Rayleigh, but he seems to have looked upon the first effect of aberration as a loss in defining power owing to spreading of the light into the immediately surrounding parts of the diffraction pattern and enlargement of the central condensation. It is of interest to speculate, therefore, what is the condition for best focus. It has been assumed in this paper that the central disc will appear relatively most bright to its surroundings when the confusion of phase at the point is least, or when the "phase residuals" are a minimum. This greatest relative brightness condition undoubtedly corresponds to the best definition, but it may be doubted whether it is the focus that would be selected in every case*. Under a low magnification (in focussing a camera lens, for example, where far greater amounts of aberration are likely) the eye may select some point where the most light is collected within a small area, giving the image which broadly possess the maximum contrast but not necessarily the best "detail definition" Is it not an astronomical experience that a planetary detail focus may not be the best for star images?

The circle of least ray confusion in the presence of aberration may correspond to a much larger patch of light than the spurious disc although, judging from the general results of the foregoing observations, the major portion of the energy is found within the limits of the bright outer ring and is not partly spread diffusely into the field as when the central concentration is at its brightest. This focus then may correspond to the "maximum contrast" focus although not to the "best definition" focus.

It is hoped that the results of this paper may help to show some of the effects

* The problem is more complicated when chromatic observation has to be considered. See paper on "Star discs" and booklet on *The Adjustment and Testing of Telescope Objectives*.

(and possibilities even) of judicious employment of spherical aberration in optical systems. Problems frequently arise in connection with photographic lenses when certain amounts of spherical aberration are allowed in the axis for the sake of flattening the field, and the result on the image of the position of the plate may be, perhaps, better understood. Some portions of the paper will be of interest to those testing optical systems, both visually or by interferometer methods.

My best thanks are due to Professor Cheshire, C.B.E., for his help and encouragement; also to Professor Conrady who originally suggested the research and to whom I am greatly indebted first for the original work which made my own possible and for his kindness in frequently discussing the subject with me. I should like to thank Professor A. Fowler, F.R.S., for reading the paper, and no less for his continued interest in the work.

DISCUSSION

Lt.-Col. Gifford asked the author which of the Seidel functions he had treated.

He thought the photographs which Mr Martin had shown were quite different from those obtained with telescope objectives. With a perfect triplet objective one obtains a white globular centre, surrounded by wire-like rings. He had no reason for doubting that the same effect would follow with microscope objectives. The effect in both cases, however, depended on the perfect squaring-on of the objective, and the author's photographs exhibited a want of correct centering.

With regard to the best height for corrections he found that for telescope objectives it was best to choose a circle which divides the areas into two equal parts. This was true for doublets, but not for triplets.

Mr T. Smith said there were many points it would be of interest to discuss, but he proposed only to mention two. The work had led to the conclusion that for definition of a high order the phase difference should lie well within Rayleigh's $\frac{1}{4}\lambda$ limit. Speaking from recollection he thought Rayleigh declared when a phase discrepancy amounting to $\frac{1}{4}\lambda$ had been introduced perceptible aberration had made its appearance. This appeared to imply that the limit was on the short side of $\frac{1}{4}\lambda$. In discussing this point Chalmers had given from $\frac{1}{8}\lambda$ to $\frac{1}{3}\lambda$ as the limits his experience suggested, and he attributed the reduction in part to the necessity for taking chromatic differences into account. This view he gathered was confirmed by the present work.

He was somewhat uncertain what other conclusions of practical interest the author drew from his investigation. Although not specifically stated it appeared to be implied that the position of best focus must be determined by investigating phase variations and that the resulting position differed from that which would follow from any theory in which the wave character was neglected, as for instance a corpuscular theory. With this conclusion he did not agree. In the paper Chalmers' method of deriving path differences from the lateral aberration had been used. The method used at the N.P.L. in which the longitudinal aberration is plotted against $1 - \cos\psi$ or $\frac{1}{2}\sin^2\psi$, where ψ is the ray inclination, offered many advantages. The effect of a change of focussing position was given in this method by the same

change in the ordinate forming the moveable boundary to the areas which measure the phase differences. It is immediately seen that the position of focus for which the phase variation is least is also the position of the centre of gravity of the ray-axis intersections on the corpuscular theory. The conclusion he drew was that the old practice of considering ray paths could not lead the computer appreciably astray. It did not of course follow that the determination of the extreme phase differences was not a most desirable calculation before the work on a system was regarded as complete. For this purpose the graphical integration method by square counting was more than sufficiently accurate and incomparably more simple than other methods which he knew.

It may be observed that when the variation in path differences is adequately represented by the inverse working distance law, the coefficient which determines the magnitude of the change may be written down at once. Taking for instance a marginal ray inclined at an angle ψ to the axis, and denoting by ρ the length on this ray intercepted between the axis and the aperture, the phase difference at a distance ξ from the intersection point is

$$\text{constant} + \sqrt{\rho^2 + 2\rho\xi\cos\psi + \xi^2} - \rho - \xi.$$

The ordinary expansion in series form is

$$\text{constant} - \xi(1 - \cos\psi) + \frac{\xi^2 \sin^2\psi}{2\rho} - \ldots,$$

but an alternative form is

$$\text{constant} - \frac{\rho\xi(1-\cos\psi)}{\rho+\xi} - \frac{\rho^2\xi^2(1-\cos\psi)^2}{2(\rho+\xi)^3} \cdots$$

or $$\text{constant} - \rho(1-\cos\psi) + \frac{\rho^2(1-\cos\psi)}{\rho+\xi} - \frac{\rho^2\xi^2(1-\cos\psi)^2}{2(\rho+\xi)^3} \cdots$$

and $-\rho(1-\cos\psi)$ may be absorbed into the constant term. Since the numerator of the term in ξ^2 contains the fourth power of the inclination in the latter expansion instead of the second as in the usual form, $\frac{\rho^2(1-\cos\psi)}{\rho+\xi}$ gives a closer approximation than $-\xi(1-\cos\psi)$ to the variation. The exact value lies between these two approximations.

He was somewhat puzzled by the claim that special modification of the Hartmann method had been devised. From the description the method was entirely normal. Whether the light coming to a focus had passed through a microscope objective, a photographic lens, or a telescope object glass was immaterial. The fact that its most frequent application had been to telescope objectives resulted from the accident that in these instances simpler methods presented unusual difficulties.

The author communicated the following reply:

In reply to Col. Gifford the experimental work described in the present paper relates to "spherical aberration" only. I feel sure that he will appreciate the point if I say that I have not "treated" any Seidel functions at all. The condition $S_1 = 0$ is an equation which says in effect: "Fulfil certain conditions in the construction of an optical system and you will ensure that the coefficient of a definite term in a

mathematical series expressing the spherical aberration of a system is zero." My work concerns a definite lens with a definite departure from sphericity in the wave surfaces it produces and consequent definite effects near the focus.

The appearance of the rings differs greatly under various conditions of observation, magnification, etc., while the photographs do not necessarily give an exact rendering of the visual appearance. Theoretical results for the light distribution in the star image produced by a well corrected lens would not lead one to infer the "wire-like" appearance of the rings. The explanation however will probably be found in the fact that Col. Gifford's star was seen with white light, while my photographs were taken with light of a narrow range of wave length only.

With regard to the centering of the objective this was unfortunate of course but even this lens was only used after having been returned for re-centering to the makers and was better centered than the majority of microscope objectives. The main reason for using it instead of another was that it was the only one I could procure with available constructional data.

In reply to Mr T. Smith no very startling results were anticipated from this work but as a matter of scientific method I hold it important that theoretical investigations should be accompanied by practical observations. My objects are set forth on page 65 lines 10 and 11: "Experimental investigation is necessary to prepare the way for full numerical work and to assess its likely value as applied to practice." It should be remembered that this is only part of a larger scheme of work, but I regret that the measurements on the microscope objective and the tracing of the changes near the focus of the lens were not found by Mr Smith to be of "practical interest." Naturally I differ entirely on this point, otherwise the work would not have been undertaken.

The method of graphical integration described by Mr Smith is interesting, but I think he would not claim that it offers any advantage in the present case, especially as the lateral aberration diagram gives a clear indication of the relation between the phase differences and the diameters of the circles of confusion. From the obvious equal spacing of the focussing points of successive zones enclosing equal parts of the area of the wave surface when simple spherical aberration is present the location of the centre of gravity of the ray intersection half way between axial and marginal foci is easy to see without special constructions, but it is by no means clear why this "centre of gravity" should have anything to do with the position of maximum light intensity, whereas it is an entirely legitimate guess on physical grounds that this maximum *will not be far from the position involving least phase residuals*. Numerical integration in fact places it a short but definite distance away from this point. As far as other theories go "the circle of least confusion" has to my mind a far better claim to be considered than the centre of gravity.

The phase aberration is certainly obtained most easily by graphical methods when the focussing points of *several* zones are known, but it is often unsafe to assume a particular type of spherical aberration in order to integrate. In this case "other methods" are more legitimate.

Mr Smith objects to my use of the term "modification" in the synopsis (I use

"adaptation" in the text). Murray's *Dictionary* gives the philosophical meaning of "modification" (verb) as "The bringing of a thing into a particular mode of existence" or (noun) "The action of making changes in an object *without altering its essential nature or character*." Was not the Hartmann test always understood to be a test devised for telescope objectives or camera lenses and conducted with a more or less distant object or auxiliary collimator? To bring a test into a mode of existence as applied to microscope objectives seems to fulfil the above idea of a "modification." As far as I know the application to microscope objects has not previously been carried out, in spite of repeated calls for a reliable method of investigating aberrations which could be applied to the comparison of different types of objective. I have "claimed" nothing epoch-making; if an incidental in the work happens to lead to a useful method so much the better.

I am at a loss to understand his concluding statement: "The fact that its most frequent application," etc.!

Returning finally to the question of aberrations expressed in terms of phase differences it cannot be too often insisted that, expressed in any other way, aberration is relatively meaningless from a physical point of view, but the most elementary student can understand tolerances expressed in terms of phase. Five years' work and teaching at the Imperial College has enabled *that* to be stated with certainty.

AN AUTO STROBOSCOPE AND AN INCANDESCENT COLOUR TOP

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Physicist to St Bartholomew's Hospital, London.

(Communicated by Professor Cheshire.)

MS. received, 21*st October*, 1921. *Read and discussed*, 8*th December*, 1921.

ABSTRACT. A description is given of some novel optical phenomena which may be observed when revolving incandescent filaments are viewed.

The first portion of the paper deals with objective phenomena and the latter portion with subjective phenomena.

In Part I the production of a variety of stationary dark images, due to the eclipse of an incandescent wire by an adjacent cold wire or opaque object, when both are revolving round a common axis, is described.

It is shown how the phenomena may be practically applied to the study of the behaviour of a rotating body, by converting such a body into an Auto Stroboscope.

In Part II a description is given of the use of an incandescent electric lamp as a Colour Top and for demonstrating "after-image" effects.

PART I. AN AUTO STROBOSCOPE

During the course of an investigation which is not yet completed on the effect of temperature on Æolian tones, the writer observed some interesting optical phenomena, a description of which it is the object of the present communication to record.

The apparatus employed was a modification of that used by Strouhal in his researches on Æolian tones. It consisted of two horizontal wooden arms rigidly attached at their centres to a vertical steel rod which was caused to revolve by coupling it to a small electric motor. The wires under investigation were stretched vertically between the extremities of the upper and lower horizontal arms and could be electrically heated by current introduced through two slip rings on the vertical shaft, and suitable conductors embedded in the arms (see Fig. 1).

On rapidly rotating a single filament heated to incandescence, a beautiful curtain of fire is seen, which calls for no special comment. If, however, in addition to the heated filament a similar but unheated wire is fixed parallel and close to it, then a dark line, curved in three dimensions, is seen projected in one half of the curtain of fire. The dark line is seen in that half of the figure in which the cold wire is nearer to the observer than the hot wire; thus by having a single hot wire at *each* end of a horizontal arm, with a cold wire in front of one and behind the other, dark lines are seen in each half of the figure. Multiple dark lines may be obtained by fixing several cold wires in proximity to an incandescent wire.

Since in the above experiments the wires are in continuous vibration in a plane transverse to the direction in which they are travelling at any instant, owing to the turbulent motion of the air through which they pass, the observations were repeated under conditions in which the transverse vibrations were not maintained. This was easily accomplished by rotating the wires in vacuo, the necessary experimental modifications merely consisting in the attachment of an ordinary commercial metal filament lamp to the armature of the motor, in place of the original framework. Under these circumstances the cold supporting wires are easily seen, apparently remaining stationary in space. A close inspection also reveals the presence of numerous stationary dark lines approximately parallel to the incandescent wires in the lamp.

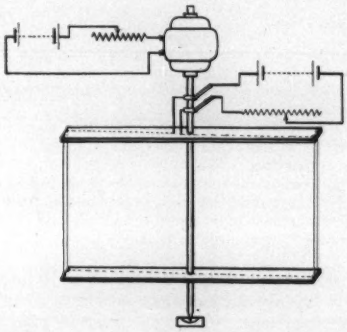


Fig. 1

It is easily seen from the above that the dark lines seen in these experiments are due to the eclipse of an incandescent wire by another hot or cold wire. If the occulting wire is not parallel to the incandescent wire or if the latter is not straight, a total eclipse of the incandescent wire does not occur at any given instant, but successive portions are eclipsed at different phases of a complete rotation of the system, and a three dimensional curve is seen which exaggerates any irregular feature of the occulted wire. The stationary appearance of the dark curves is a "persistence of vision" phenomenon due to the periodic variation of the intensity of the light falling on the eye of an observer looking in a given direction.

A somewhat surprising effect was noticed when various incandescent lamps were set spinning. It was observed that the manufacturer's name and other letters and figures etched on the glass envelope, could easily be read while the lamp was spinning at all speeds from 5 to 20 revolutions per second (the maximum speed attainable with the given arrangement). A number of these images could be seen at the same time, some being erect and others inverted. They differed from the markings on the glass envelope in that the images appeared black, whereas the etched figures were white.

That this effect is of the same kind as the eclipse effects described above, can readily be shown as follows:—

A single filament lamp, described commercially as a "Linolite" or "Stryplite," is laid on a circular whirling table and securely attached with the filament lying along a diameter (see Fig. 2). On slowly rotating the table, the lettering on the

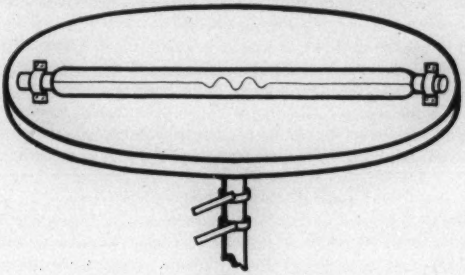


Fig. 2

anterior surface of the glass envelope is seen to eclipse the glowing filament twice during a complete revolution, once in the erect position, and once with the letters laterally inverted. Rapidly spinning the table gives the erect and inverted dark images at opposite ends of a diameter. A similar result is obtained, but with bright instead of dark images, by fixing a stencil plate to the table just above the filament. An example of this is shown in the photograph (see Fig. 3).

Fig. 3. Photograph of a number (5 6) on a metal stencil plate rigidly attached to an incandescent lamp and rotating at a speed of 450 R.P.M. Exposure of 2 minutes.

It is obvious from the above experiments that the contours of any rapidly rotating object can be viewed very easily by attaching to it an incandescent filament (or a brightly illuminated line on a black background), provided the object is placed at a sufficient distance in front of the filament that occultation can be seen to take place during a revolution.

Such an arrangement, which gives a stroboscopic effect without the aid of an independent moving part, constitutes what may be termed an *Auto Stroboscope*.

An auto stroboscope possesses a number of advantages over most of the usual stroboscopic devices, in that it

(1) maintains synchronism at all speeds without the necessity for subsidiary adjustments,

(2) is simple to fit up,

(3) can be used in daylight, as well as in a dark room,

(4) can be adjusted to give a magnified image of any irregularity of shape of the rotating object by altering the position of the incandescent filament (or luminous line) relative to the object,

(5) is well adapted for the taking of photographic records.

It possesses, however, the disadvantage of giving only contour images.

PART II. AN INCANDESCENT COLOUR TOP

The experimental arrangements described above are also suitable for the study of certain aspects of colour vision and related psycho-physiological effects.

Various methods of exciting different colour sensations by rotating discs of coloured pigments have been described and are in common use. For example, the colour discs and tops of Helmholtz, Maxwell, Benham, Sherrington, etc. are well known. Other devices are also described in books on Physiological Optics*.

As a result of spinning an electric lamp whilst its filament was incandescent, the writer discovered that it behaved as a novel colour top. Both carbon and metallic filament lamps are suitable for demonstrating this, but the former usually withstand the somewhat rough usage better than the latter. Similar effects may also be obtained by using bare wires (preferably of platinum) heated in the open air. In this case the luminosity for a given applied voltage will vary with the speed of rotation, and care has to be taken that the same speed of translation through the air is imposed on all parts of the filament in order to prevent irregular convective cooling, and consequent fusion of the hotter, slower moving parts.

When using an ordinary 200 volt lamp connected to the *continuous current* mains, beautiful colour effects are obtained if the lamp is rapidly rotated about its own axis of symmetry. The colours which are seen vary with the speed of rotation, the luminosity of the filaments, and the position of the observer, and can best be observed in a darkened room.

The phenomena can be studied more simply by using the arrangement and lamp shown in Fig. 2. Using this form of apparatus and regulating the current through the lamp so that the filament has a decidedly yellow hue, various hues of blue and yellow-green can be seen at low speeds (4–5 revs. per sec.) whilst at higher speeds reds make their appearance, and practically all the conventional spectrum colours can be seen at suitable speeds and filament temperatures. The general appearance is that of "shot silk," accompanied by bright and dark radial streaks.

* Vide Parsons' *Introduction to the Theory of Colour Vision*, Cambridge 1915, for a list of references.

Bright filaments demand higher speeds of rotation to give colour effects than do those of lower luminosity.

At very low speeds of rotation (one revolution in one or more seconds) well marked "positive after image" effects may be viewed, the straight bright filament having the appearance of a luminous propeller. At slightly higher speeds the outer edge of the luminous pattern is interrupted by dark bands—the so-called Charpentier bands. The radial streaks mentioned above are probably examples of the same phenomena.

The preceding experiments on the production of colour sensations by the rotation of a nearly white light simulate in some ways the analysis of white light into a spectrum by a prism.

It is perhaps worthy of record that the converse phenomenon of the synthesis of white light from coloured lights may also be imitated by spinning a lamp giving a coloured light. A suitable lamp for this purpose is a "Cathode glow" lamp, containing neon, helium and mercury vapour*. A lamp of this kind gave when stationary a central pink glow, surrounded by a blue halo. When rotated rapidly about its axis of symmetry it gave other colours in addition to its stationary colours, and also a band of white light.

The same lamp produced a remarkable after image effect when attached by a universal (ball and socket) coupling to the armature of the motor, and set spinning eccentrically. When rotating at a speed of less than one revolution per second, a positive after image of the blue halo, which appeared to extend over 180 degrees, was seen.

It is not the object of this paper to discuss the physiological and psychological interpretation of the colour vision phenomena here described, but rather to call attention to the novel way in which they are produced, and their great variety.

In conclusion, I wish to express my thanks to my assistant, Mr F. C. Bratt, for considerable experimental assistance in carrying out the observations described in this paper.

DISCUSSION

Mr J. Guild said the author was to be congratulated on having discovered such a simple and effective way of exhibiting the colour effects observed with moving sources of light. These effects were not due to fatigue, as had been suggested, but to differences in the time lag which precedes perception of a visual sensation. H. E. Ives† showed some years ago that the phenomena of intermittent vision could all be accounted for on the assumption that a visual message passing from the retina to the brain obeys laws analogous to those governing the conduction of heat or electricity, the transmitting medium having a coefficient of *diffusivity* which varies with the wave length and intensity of the light. The time lag between the receipt of a stimulus and its perception is greater for blue than for red light and,

* A lamp of this kind, which can be run off a 200 volt supply, is sold by the General Electric Co., Kingsway, London.

† *Phil. Mag.* XXXIII. (1917) 18.

for any wave length, is greater for low than for high illuminations. This, coupled with the fact that the differently coloured images are drawn out into a fan by persistence of vision, and so partially superposed, accounts for the colours observed in the author's demonstration. The predominant colour depends on the relation between the difference in the lag for different colours and the time interval which elapses before any part of the retina is re-exposed to the stimulus, that is, on the speed of rotation. The same explanation applies to the colour shown by Benham's colour top, and the colours which may be observed on looking at a white surface through a sector disc rotating at suitable speeds.

Mr T. Smith said it was conceivable that a pretty toy, such as the author had exhibited, might in the future be of considerable value.

He begged Dr Hopwood not to exploit the thing for advertisement purposes, even if there was money to be made with it.

Prof. Cheshire pointed out that colour decrement curves are shown in Nutting's *Outlines of Applied Optics*.

Lt.-Col. Gifford thought that the author had clearly demonstrated that retinal fatigue exhibits a differential effect as regards colour.

Mr M. Blood said that, in connection with the suggestion of a previous speaker that the rate of nerve transmission varies with the strength of the stimulus, he had an extraordinary experience during the war. He had occasion to make an experiment of firing a very high velocity rifle at a hard steel plate some ten yards away. Portions of the bullet rebounded and cut him on the face, and the sensation was very strong that he had been struck an instant *before* he pulled the trigger! The difference between the intensities of the two impulses was great and hence presumably between the rates of transmission of these to the brain.

The President asked if Dr Hopwood had tried the effect of twisting or zigzagging the cold wires.

The author, in reply to Mr Guild, stated that he did not presume to be an expert in colour theory. The only novelty he claimed for the colour top was the rotation of the source of light instead of the illuminated surface. In ordinary stroboscopic arrangements auxiliary mechanism, which had to be maintained approximately in synchronism, is usually employed. The effects he obtained were stroboscopic at all speeds at which one is getting persistence of vision.

In reply to the President, he stated that exaggerated curves can be obtained by twisting or inclining the wires.

ACHROMATIC ONE-RADIUS DOUBLET EYEPIECES

By Lt.-Col. J. W. GIFFORD, F.R.A.S.

MS. received, 7th November, 1921. Read and discussed, 8th December, 1921.

In May, 1920, I placed before the Optical Society details of a short high power telescope composed of one-radius achromatic doublets with external plane surfaces to the flint lenses. Since that time eyepieces both of the Huygenian and the Ramsden types have been constructed from pairs of such achromatic doublets. They compare well with the German orthoscopes in definition, while the cost of production, since the same radius serves for each doublet, or in the case of the Ramsden throughout, is sensibly less. Such eyepieces are adapted either for the telescope or the microscope. By their use a much more perfect achromatism is obtained and also in both of them a flat field, and in one of them a very extensive flat field, likely to be useful in such operations as counting blood corpuscles, etc.

(1) *Huygens' type.*

The only claim which this eyepiece possesses to be so called is that all air surfaces are in the same order as in the Huygenian, that is, the first is convex, the second plane, the third convex, and the last plane (Fig. 1). The field lens doublet has twice the focal length of the eye lens doublet. Both field and eye lenses are one-radius doublets in which the conditions for achromatism are satisfied. This eyepiece resembles a micro-objective reversed.

The glass meltings used are Schott's No. 3581 (type O. 3390) boro-silicate crown and Chance's No. 1794 (type 370) heavy flint, with refractive indices for line A of 1·51912 and 1·62160 respectively and a ratio of mean dispersion

$$= \frac{\delta\mu'}{\delta\mu} = 2{\cdot}002^{*}.$$

Other particulars are as follows:

	Field lens	Eye lens
Diameter ...	1·00	0·75
Radius of curvature	1·000	0·500
Crown thickness	0·280	0·350
Flint thickness	0·120	0·060

Separation	...	0·576	Focal length	...	1·000
Gauss *D*	...	0·821	Eyepoint	...	0·692

* A fully satisfied achromatic one-radius doublet can alone be obtained when the ratio of relative dispersions is 2, in other words $\frac{\delta\mu'}{\delta\mu} = 2{\cdot}00$.

(2) *Ramsden type.*

Everyone knows that originally Ramsden's idea for achromatism was that two simple plano-convex lenses of equal focus should be so arranged that $f = f' = D$, where f is the focal length of the field lens, f' that of the eye lens, and D their Gauss distance apart, and that these simple single lenses should, for the purpose of aplanatism have their convex surfaces in opposition and should be placed one in the focus of the other (Fig. 2). But this ideal construction has not been realised. It was found that in order to use the combination the eye must be placed in contact with the outer (plane) surface of the eye lens, a thing impossible in practice. In order to get enough eye room therefore, d the actual distance between the lenses had to be something less than f or f', and in practice it became anything between $\frac{3}{8}f$ and $\frac{3}{4}f$ with a corresponding incomplete correction for the so-called achromatism.

Fig. 1

Fig. 2

Fig. 3

Now if we compute a one-radius doublet from certain ordinary flint and crown glasses already referred to of mean dispersions ($\delta\mu$) 0·0170 and 0·0085 we shall get $\frac{\delta\mu}{\delta\mu'} = \frac{0{\cdot}0170}{0{\cdot}0085} = 2{\cdot}0$, a condition of achromatism far removed in completeness from Ramsden's. Let F_1, F_2 be the equivalent focal lengths of two such combinations and let $D = 0{\cdot}20\,(F_1 + F_2)$. We have then constructed our new Ramsden eyepiece (Fig. 3), an achromatic combination with a sufficiency of eye room and with its principal planes coincident. In practice the eye lens need only be half the diameter of the field lens. With the same glasses as before the data are:

	Field lens	Eye lens
Diameter ...	0·675	0·375
Radius of curvature	0·681	0·681
Crown thickness	0·225	0·113
Flint thickness	0·090	0·060 or 0·045

Separation ... 0·693 Focal length ... 1·000

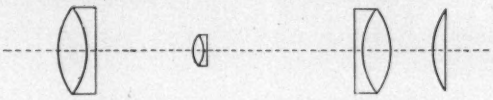
Note. Either of the alternative figures for the flint thickness of the eye lens may be taken, by preference the higher value when the focal length of the whole combination is less than, the lower when it exceeds, 0·50. The Gauss principal planes are then respectively 0·0180 and 0·0121 apart and the eye space 0·469 and 0·478 from the outer surface of the eye lens. The apparent field is about 35°.

Three of these one-radius doublet Ramsdens of different powers have already been constructed of 14, 8·5 and 7 mm. focal length respectively by Mr C. Baker, the equivalents in inches being 0·5512, 0·3346 and 0·2756. I again have to thank Mr Nelson for valuable suggestions and for his incomparable critical help. With his consent, in issuing the eyepiece, I have associated his name with mine.

Additional Note received, 18th January, 1922, read, 9th February, 1922.

In the discussion which followed the paper other types of eyepiece were referred to; one of them of the Ramsden kind, No. 4 (see below), was then under construction by Messrs James Swift and Son, Ltd. This one, since completed, has turned out well. Another, No. 3, constructed last year by Mr C. Baker, is of the Huygenian type. Both these give a flat field with the microscope and a 160 mm. tube. Those previously described were designed for the long tube. No. 4 is undercorrected and is therefore, perhaps, more suited to the telescope. But by substituting Messrs Chance Bros. type 646 boro-silicate crown for Schotts' type O. 3390 we get $\frac{\delta\mu'}{\delta\mu} = 3{\cdot}129$ as against 2·945 given below, and the eyepiece is overcorrected without seriously upsetting the focus, and is well suited for the microscope with apochromatic objectives. One pair of tools only is required for No. 4.

Since the refractive indices of the glasses used for these eyepieces probably have errors in the sixth figure only, and in view of multiplication by the focus required, a greater number of figures for the data may be desired. These can always be obtained by addressing to the writer at "Oaklands," Chard.



No. 3 No. 4

(3) *Huygenian.* (2 doublets.)

	Field lens	Eye lens
Glasses	Schotts' boro-silicate crown, type O. 3390	Same as Field lens
	Chance's heavy flint, type 370	Same as Field lens
Achromatism (thin lens)	$\frac{\delta\mu'}{\delta\mu} = 2{\cdot}005$	
Diameter	1·000	0·367
Radius of curvature	0·860	0·287
Crown thickness ...	0·325	0·108
Flint thickness ...	0·125	0·0416
Field 60°	Separation 1·013	Focal length 1·000
	Gauss D 1·333	Eyepoint 0·398

(4) *Ramsden.* (1 doublet, 1 single lens.)

	Field lens	Eye lens
Glasses	Mantois' very dense flint, type C. 865	Schotts' crown, type O. 3390
	Schotts' crown, type O. 3390	
Achromatism (thin lens)	$\frac{\delta\mu'}{\delta\mu} = 2{\cdot}945$	
Diameter	0·750	0·750
Radius of curvature	0·839	0·839
Crown thickness ...	0·225	0·113
Flint thickness ...	0·090	—
Field 55°	Separation 0·150	Focal length 1·000
	Gauss D 0·0582 = $(F_1 + F_2)$ 0·014	Eyepoint 0·903

Note. It is recommended to reverse No. 4 combination. Definition is then extremely sharp and there is little lost in flatness of field. It is always well to adjust the separation by experiment.

DISCUSSION

Mr T. Smith expressed the hope that other members of the Society would follow the author's example in bringing forward instruments of special construction.

He asked if Col. Gifford could give some idea as to how his eyepiece compared with others. During the war he had been struck by some of the German eyepieces, which in some cases were over-elaborated for special purposes. He considered that the less flat field in English eyepieces was compensated by their greater brilliancy.

He thought that the apparent field of 35°, mentioned in the paper, was far from meeting the more extreme requirements. For many cases a field of 50° was used, and in some cases even 65°. Theoretically, very good results could be obtained with these.

He was not quite sure that the author was making the best possible use of his materials. The eyepieces are all right as regards colour, but is the effect which produces the least amount of eye-strain when used over long periods obtainable with this type of eyepiece?

Professor Cheshire said he understood that Col. Gifford had limited himself in his construction to one-radius curves and that it by no means followed that the eyepieces described were the best that could be produced from the given glasses. From what had been said by Col. Gifford he concluded that the Ramsden modification was probably excellently adapted for micrometer work. He should like to know if this was the case.

Mr M. Blood asked how much commercial advantage there was in making the eyepiece with one radius of curvature. He had looked at the objects on view and the eyepieces seemed very good as far as could be judged off-hand. He noticed particularly the absence of the bad astigmatic errors one usually found at the edge of the field of a wide angle eyepiece.

Mr C. Lees Curtis said that only one pair of tools was required for each lens, but, owing to the steepness of the curves, only four lenses could be worked in a block.

The chief advantage of the eyepiece lay in the fact that a flat object remained in focus right up to the edge of the field.

The author, in reply to Mr Smith, said that he was not prepared to say what was the best form. He had designed another form (since described, see p. 101), making better use of the glass, which he thought would shortly be ready for trial. It consisted of three components only, one flint and two crowns.

He thought the field of the Huygens' type was about 55°.

The eyepieces would be useful for micrometric and similar work.

THE MANUFACTURE OF OPTICAL GLASS

By C. J. PEDDLE, M.B.E., D.Sc., F.I.C., F.Inst.P.

MS. received, 7th December, 1921. *Read and discussed, 12th January*, 1922.

SYNOPSIS. The history of the manufacture of Optical Glass can be divided into four epochs, Guinand's discovery of the stirring process in 1796, the work of Abbe and Schott *circa* 1882, and the development in England during the Great War being the outstanding features in this history. The method of manufacture is practically the same at the present time as in Guinand's day, any improvement being one of degree rather than of kind. Thus the glass is melted in special pots and furnaces, stirred after founding, and allowed to cool *in situ*. When cold it is broken up and examined. Defective glass is rejected, and the good glass is moulded into slabs, prisms or lenses, and finally annealed. Optical glass has to conform to a rigorous specification. It must have certain specified optical constants, must be free from striae, bubble, strain, and colour, and must be durable.

The manufacture of optical glass is based upon continuous research into the properties of glass. The effects of composition upon density, refractive index, melting properties, durability, freedom from colour, and devitrification tendencies, have to be studied upon a small scale, and the results translated into terms suitable for works practice. The whole manufacture indeed is research upon a manufacturing scale.

1. INTRODUCTION

The history of the manufacture of optical glass can be written in four chapters—(1) early attempts, (2) the period 1790–1886, (3) the period 1886–1914, (4) the period 1914 onwards. The last named period has seen great developments, particularly in this country, and it is mainly with these developments, due to the exigencies of a great war, that this paper has to deal. However, in order to present a coherent account of the manufacture through all its phases, it will not be out of place to dwell very briefly upon each of the other three periods in turn.

(1) *Early attempts.* Very little is known regarding the methods of optical glass manufacture previous to the end of the 18th century, and at all events only very small pieces were ever obtained. The chief use of optical glass was in the construction of telescopes, and the computations by Dollond, about 1753, of the correct curvatures of crown and flint lenses for telescopes led to a revival of the interest in the manufacture of pieces of glass sufficiently good for this purpose. In 1768 prizes were offered by the Society of Arts for improvements in manufacture and about 1800 a committee of the Royal Astronomical Society of London, composed of Herschel, Faraday, Dollond, and Roget, was conducting an enquiry into the manufacture of flint glass for optical purposes. In view of what will be said later it is worthy of note that this committee arrived at the conclusion that the necessary

improvement was chiefly mechanical, not chemical, and that freedom from striae could only be obtained by agitation of the molten glass.

About this time the method of manufacture seems to have been as follows: a pot of glass was melted in the usual way, using the purest materials obtainable, and the surface was cleaned by skimming. An iron ladle, shaped like a truncated cone, about 5 inches in diameter at the wide end and 7 inches deep, was dipped carefully into the molten glass and as carefully withdrawn. After cooling somewhat, the piece of glass, like a sugar-loaf in shape, was welded at the wide end to a blowing iron. The glass was then re-heated at the mouth of the pot and when sufficiently soft was blown into a cylindrical hollow piece, a method employed to-day as a preliminary to tube drawing. The ends of this cylinder were then broken off by drawing round them a cold piece of iron. The cylinder left was scratched longitudinally on the inside and placed in an annealing oven, where it spread out and sank down into a flat sheet (about 14 inches by 10 inches and half an inch thick) and was annealed. These plates were sold to the instrument maker for cutting and grinding into discs from which lenses could be made. As it was seldom that much of a plate made by this method was homogeneous, it is obvious that only small lenses could result. Two celebrated scientists of that time, Macquer Roux of St Gobain and Auut of Langres, never obtained lenses larger than 3 to $3\frac{1}{2}$ inches in diameter, whilst Dartiques, an eminent flint glass manufacturer in France, was equally unsuccessful in obtaining large discs.

(2) *Period* 1790–1886. About 1790 a Swiss watch maker, Guinand, startled the optical world by producing a flint glass lens 9 inches in diameter. He communicated his method to Utzschneider and Frauenhofer and joined them at Munich. For several years they made optical glass without divulging the secret of their method, which consisted in agitating the molten glass by stirring it with a fireclay stirrer, and then allowing it to cool down slowly. When cool the glass was broken up and the good separated from the bad. On Guinand's death in 1823, his two sons to whom he had communicated the secret continued to make optical glass. One worked at Neufchatel and the other joined Bontemps at Choisy-le-Roi. Little is known of the work at Neufchatel, but the experiments at Choisy were not a success at first, and the association between Bontemps and Guinand was broken. Bontemps however persevered without excluding Guinand and in 1828 succeeded in making discs up to 14 inches in diameter. In 1848 Bontemps, in consequence of political difficulties in France, came to England and joined Messrs Chance Bros. who became the leading firm in the manufacture of large telescope discs. In France Feil, a grandson of Guinand, pursued the manufacture and it is a matter of interest that the traditions of the house of Feil are now carried on by Messrs Parra-Mantois of Paris. During this second period (1790–1886) only two types of glass were manufactured, namely crown and flint, and of these two types very few varieties were available. Glass was usually brought under a density specification during the earlier portion of this period, but later refractive indices were demanded, and these were supplied by the manufacturer. About 1875 Messrs Chance Bros. made about six varieties of glass, a Hard Crown, Soft Crown, Light Flint, Dense Flint, Extra

Dense Flint, and Double Extra Dense Flint. It will be seen that the computer of a lens system about this time had very little choice in the glasses at his disposal, and it became evident, as other optical instruments besides telescopes were invented, that unless new glasses were discovered, scientific progress was likely to be seriously delayed. Particularly was this the case regarding the application of the microscope to scientific problems. Serious attempts were made during the earlier years of the 19th century to obtain glasses having a greater variability in the relationship between dispersion and mean refractive index than was presented by the then available Flints and Crowns. As chief amongst these attempts must be mentioned the work of Harcourt who between 1834 and 1859 made numerous small scale meltings and who did produce small amounts of new types of glass. Later Stokes made experiments, and these, although barren of practical results, indicated the specific effect of certain acids and bases in glass upon the refraction of light, and laid the foundation of later successful work along the same lines.

(3) *Period* 1886–1914. It was left to Germany to revolutionise the manufacture of optical glass by the production on a large scale of several new types, and numerous varieties of these types. The classical work of Abbe and Schott at Jena is now too well known to need more than a passing reference. By patient perseverance and refusal to be deterred by failures they put at the service of the optician such new types of glass as Barium Crown, Barium Flint, Zinc Crown, Boro-silicate Crown, Phosphate Crown and Borate Flint. Later the Jena works produced Fluor Crowns and Telescope Flints as well as new glasses very transparent to ultra violet light and also new glasses transmitting only certain wave lengths of light. The manufacture of these new types at Jena led to a revival of interest in the optical glass industry in both England and France, and we find that in 1914 Messrs Chance Bros. and Messrs Parra-Mantois listed practically all the new types of glass first developed in Germany.

It should be understood that the method of actual manufacture in 1914, and at the present time, is essentially the same as in Guinand's day. Any improvement in manufacture regarding the melting and preparation of the glass has been one of degree rather than of kind. Thus melting furnaces have improved beyond measure, the old wood, charcoal, or coal burning furnace having been replaced by one burning producer gas or even town gas. Machine operated stirring has replaced the old laborious method of hand stirring, whilst improvements have taken place in methods of moulding and annealing the glass.

(4) *Period since* 1914. The outbreak of war in 1914 found this country ill prepared from the point of view of optical glass supply. Previous to 1914 some 60 per cent. had been imported from Germany and 30 per cent. from France, so that when war was declared on Germany practically 90 per cent. of our supply was stopped, because France required all her output for herself and Russia. Fortunate it was for England that Messrs Chance Bros. were in active operation in 1914, and they bore alone the whole brunt of production for more than two years. In 1916 the great advances in aerial photography and the large increase in *personnel* in our fighting forces caused the demand for optical glass to be multiplied many times

and the Government began to look for other sources of supply. Accordingly Messrs Wood Bros. Glass Co. Ltd. decided to attempt the manufacture at their Derby Works. Begun in June, 1916, before the end of the war, all the main types of glass in demand had been manufactured on a large scale, some 70 different varieties of optical glass being put at the service of the optical instrument manufacturer. Since that time several further varieties have been made, so that at present more than 100 different glasses are manufactured at Derby.

2. METHODS OF MANUFACTURE

In the manufacture of optical glass there are many points to be taken into account, and some of these may be set out as follows:

1. Each glass must have certain refractive indices for various wave lengths of light, and successive melts of the same type of glass must only vary within narrow limits in their respective indices.

2. The glass must be as free from bubble as possible.

3. No veins must be present. Absence of veins or striae, visible to the naked eye alone, is not sufficient, and glass has to be examined on a special instrument for the presence of "fine striae."

4. Even in pieces 1 or 2 inches thick there must be practically no colour. In addition, freedom from colour must be brought about without use of "decolorisers," such as manganese dioxide, nickel oxide, or selenium, because the glass must transmit a maximum of light.

5. It is essential that the glass shall be very durable, that is, it must preserve its polished surface undimmed and without disintegration after long usage under all climatic conditions.

6. The glass must be free from strain, and so demands a high degree of annealing, a particularly delicate operation with thick pieces of glass.

7. The working properties of the glass must be such that it can be easily fashioned into lenses and prisms. It must not be too hard nor too brittle, so that it may be ground and polished.

8. The batch mixture must be such as to admit of easy founding under normal glass melting conditions, and must yield a glass of sufficiently low viscosity, when molten, to admit of easy stirring.

9. After stirring, the pot of optical glass is allowed to cool down *in situ*, and is broken up when cold. This cooling must be at a fairly slow rate (a matter of days) so that fairly large pieces of glass are available. Slow cooling introduces another pitfall into the operation, namely, the separation from the glass of one or more of its constituents, leading to "devitrification." Thus care has to be exercised that composition and rate of cooling are such that no separation takes place.

It will be seen that many difficulties stand in the way of the production of first rate optical glass, and the reason will be appreciated why often less than 10 per cent. of the glass melted is saleable. Perhaps the greatest difficulty, and one not mentioned above, is the pot in which the glass is melted. In the writer's opinion only this pot difficulty stands between the optician and glass of a quality hitherto unattainable

either in quality or price. Molten glass has a very corrosive action upon the clay of which glass melting pots are made. This results in the absorption by the glass of iron, alumina, and silica. Iron absorption is the cause of the colour in optical glass, and the incorporation of alumina and silica largely accounts for the striæ and at the same time increases the difficulty in obtaining the desired optical constants of the glass melted. If only a suitable refractory could be obtained upon which glass, when molten, had little or no action, many of the difficulties of glass melting would disappear. It must be said in passing that the search for such a refractory is incessant at Derby. Considerable time and much money have been spent upon it, and improvements have been introduced.

Having outlined some of the difficulties it is necessary to show how an endeavour was made to overcome them, for it is obvious that no progress in manufacture could be made unless a considerable amount of research were carried out upon each point mentioned. Perhaps the simplest method will be to give an account of the procedure followed in the works in obtaining a specific glass ordered by a manufacturing optician, assuming that no sample of a like glass is obtainable. The following steps in the manufacture are necessary: (1) preliminary experimental work, (2) batch mixing, (3) melting, (4) stirring, (5) cooling, (6) breaking up and examination, (7) moulding and rough annealing, (8) polishing and examination, (9) fine annealing, (10) investigation of optical properties.

Preliminary Experimental Work. When ordering his glass the optician specifies certain refractive indices, usually for the *C*, *D*, *F* and *G'* lines of the spectrum. In order to get these optical constants within the limits allowed (which are usually $\pm$ 0·001 in the *D* line and $\pm$ 0·1 in the ν value) it is necessary to incorporate the proper batch making materials in correct proportions. Since 1916 the author has been conducting researches upon the relationship between the composition of a glass and its optical properties, and proposes to refer more fully to these researches later. From the data obtained it is usually possible to choose a composition very close to the one desired for the glass required. Small melts are then carried out in the laboratory, using this composition. Experience has shown that the optical constants obtained from these small melts are not the same as would be obtained by melting the same mixture on a large scale. They are always lower, due to the fact that attack on the pot is greater per unit weight of glass melted in the small pot. Thus more silica is absorbed in the small melting, resulting in lower refractive index. Sufficient knowledge, however, is now available to translate the results obtained on small scale melting into terms applicable to melts upon a works scale. The refractive indices of the small melts are taken and, if unsatisfactory, new melts are carried out, using different batch proportions, until a correct batch is obtained. These small melts are also used to obtain information as to the durability of the glass, its annealing temperature, tendency to devitrification, etc. Information is also obtained regarding the melting properties and viscosity of the glass from the small melts.

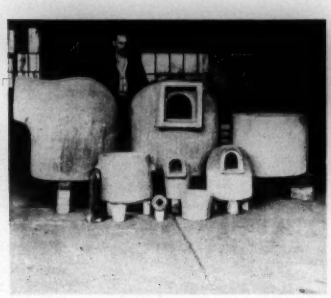
Batch Mixing. When a mixture has been determined, it is necessary to prepare it upon a large scale. As a rule the amount made up varies from 10 to 15 cwts.

according to the type of glass melted and the size of the melting pot. A prepared mixture is known as a batch. The batch for a flint glass is obtained by mixing together in the requisite proportions sand, red lead oxide, sodium carbonate, potassium carbonate, and potassium nitrate. Thus the analysis of a flint glass can be stated in terms of the oxides of silica, lead, sodium, and potassium. The batch for a crown glass contains calcium carbonate in place of red lead oxide. Barium crown batches are composed chiefly of sand, barium carbonate, zinc oxide, alumina, and borax or boric acid. Barium flint batches contain red lead oxide in addition to the other ingredients of a barium crown batch. All materials for optical glass melting are obtained in as pure a condition as possible, special care being taken to have them as free as possible from iron compounds. These materials are dried and sieved and then thoroughly mixed in correct proportions, the mixing being done in a revolving drum.

Melting. The batch is stored where it will not absorb moisture, and is melted as soon as possible after mixing. This melting is carried out in special pots about 3 feet high and 3 feet in diameter, the walls being 3 or 4 inches thick. The depth of molten metal is about 18 to 20 inches. As a rule optical glass pots are covered and the mouth of the pot projects well away from the body, to prevent the entry of deleterious material. Photograph 1 shows several sizes and shapes of glass melting pots. The two large pots, one shown front view and the other side view, indicate the size of the pot generally used for large scale working. Pots for melting glass are made from a very refractory fireclay; their manufacture is an industry in itself and cannot be entered into here. The furnaces in which the pots are heated vary in constructional details, and space forbids any discussion of them. Further information regarding pots and melting furnaces can be obtained from literature on the matter, a considerable amount of which has been published recently (see appendix).

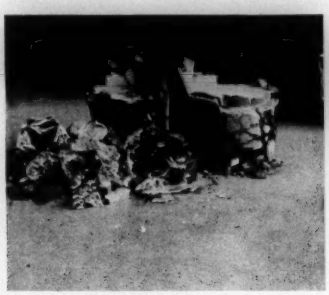
As a rule furnaces are built to accommodate one melt at a time, because each melt requires special treatment from the point of view of temperature, time of founding, etc., and the single unit furnace permits of exact control. Modern furnaces are heated by gas, either generated from producers or taken from the town's mains, the latter system being the less troublesome but the more expensive. Photograph 2 shows a modern melting furnace for optical glass production.

Before placing in the furnaces, the melting pots have to be heated up to furnace temperature. The pot arrives at the works in a dry state, but unburnt, and being very brittle requires careful treatment. It is placed in a "warming up" chamber known as a "pot arch" the temperature of which is gradually raised at a regular rate until after several days it is about 1000° C. The pot is then at a good red heat, and ready for placing in the furnace, which is also at about 1000° C. The removal of the pot from the "pot arch" to the furnace is a very onerous operation requiring great care to prevent breakage. After the pot is placed in the furnace the temperature is raised and the batch is filled in. The progress of the melt is carefully followed and the temperature regulated according to the type of glass being melted. Some glasses, such as boro-silicate crowns and hard crowns, may require temperatures as

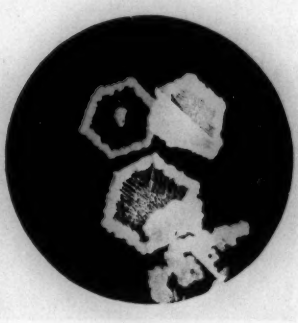


Photograph 1.

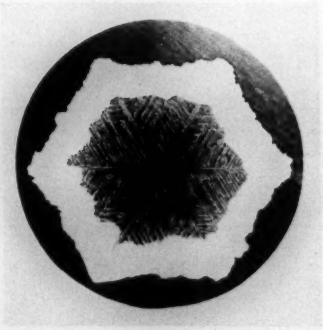
Photograph 2.



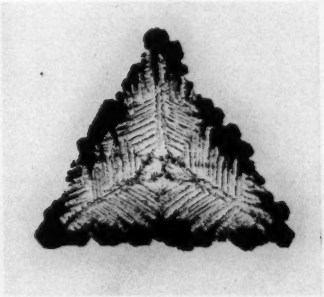
Photograph 3.



Photograph 4. Magnification 21 diameters



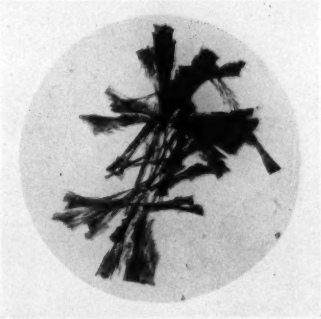
Photograph 5. Magnification 58 diameters



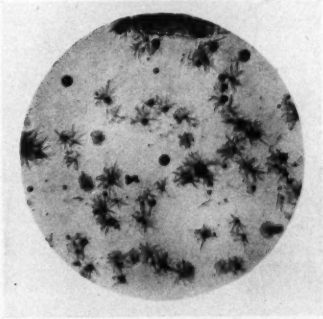
Photograph 6. Magnification 58 diameters

Photograph 7. Magnification 25 diameters

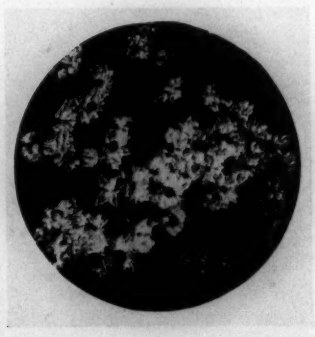
Photograph 8. Magnification 8 diameters



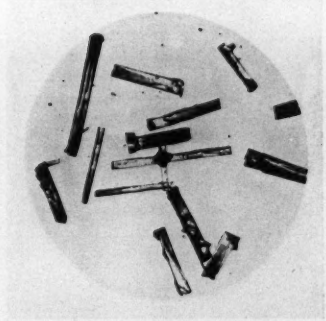
Photograph 9. Magnification 20 diameters



Photograph 10. Magnification 8 diameters



Photograph 11. Magnification 8 diameters



Photograph 12. Magnification 6 diameters

high as 1500° C. Flints on the other hand are more easily melted, the denser types requiring temperatures little removed from 1000° C. The time of "founding," i.e. transforming the powdery batch mixture into the vitreous molten material known as glass, varies, being on an average 20 hours. It should be mentioned here that in some glass melts broken glass from previous melts of similar type is melted up with the batch. This broken glass is known as "cullet" and its incorporation with the batch mixture expedites the founding. When the "founding" is finished no batch is left, but the molten vitreous mass is filled with gas bubbles. The removal of these is known as "fining" and generally takes several hours and necessitates temperature changes. In certain glasses, particularly dense barium crowns, it is almost impossible to remove all the gas bubbles. After "fining" the surface of the glass is skimmed to remove any floating impurities. It is then ready for stirring.

Stirring. A fireclay or porcelain cylinder which has been previously heated to the furnace temperature is introduced into the molten glass. This cylinder is hollowed out, and into this hollow is fitted a water cooled stirring rod. The rod is attached to a stirring machine (a motor driven crank) which when set in action moves the fireclay cylinder round and round the pot. The stirring continues until the rod can only be moved with difficulty. The machine is then stopped, the rod removed, and the pot allowed to cool down *in situ* or in a suitable cooling chamber. Stirring machines are of various designs and admirable descriptions with diagrams and photographs will be found in technical papers on glass (see appendix). Two stirring rods are shown in Photograph 1.

Cooling. The cooling of a pot of optical glass is an art in itself and requires careful control. Rapid cooling immediately after stirring is useful, as it serves to prevent after action and movement of the glass by convection, both of which set up striae. Below a certain temperature, which varies according to the type of glass, much slower cooling is necessary, so that the glass when cold is not unduly strained. If the glass when cold is badly strained it flies into small useless pieces, and it cannot be broken up. Slow cooling, however, has its pitfalls, and if unduly prolonged between certain temperature ranges may cause "devitrification" of the glass, i.e. a separation of some constituent in an opaque form. Further reference to this phenomenon of "devitrification" will be made later. The cooling of an optical pot is a matter of days.

Breaking up and Examination. When the pot is cold, the hood is removed and the pot is broken open. The glass is found to be cracked in many places. If the cooling has been satisfactory, fairly large lumps, reasonably rectangular in shape, are obtained. Photograph 3 shows pots broken open. The lumps after breaking are examined for defects such as striae, stones, and bad bubbles. Defective portions are broken off and the remainder is ready for the moulding process. At this stage at least 50 per cent. of the glass is rejected even in a melt considered satisfactory.

Moulding. The pieces of glass prepared as above are put into suitably sized square moulds and placed in a moulding furnace. This is simply a long tunnel with one end heated. The glass is placed at the cool end and gradually moved down the tunnel until the hot end is reached. This method allows the glass to become hot

without cracking, until at last it flows down and fills the mould. The square slabs so formed are then allowed to cool down slowly to "rough anneal" them and they are then ready for the second examination process.

Second Examination. The slabs are taken and opposite ends are ground by carborundum and polished with rouge. They are then examined with the naked eye for obvious defects such as striae, moulding faults (feathers), stones, and large or excessive bubbles. Defective pieces are rejected. The remainder are then subjected to a much more stringent test for striae by means of a polariscope. This instrument shows up striae which are invisible to the naked eye, but which render the glass unfit for first class optical work. The glass of first quality left after the examination is then ready for fine annealing, and if the amount is about 20 per cent. of the original glass melted the yield is considered satisfactory. More often than not this percentage is not obtained.

Fine Annealing. The glass is next subjected to the process of "fine" or "optical" annealing, which consists in gradually raising the glass in temperature almost to deformation point and then allowing it to cool slowly over a period of several days. The most modern fine annealing plant is simply a well insulated box made of refractory material and heated electrically. Perfect control can then be exercised over the rates of heating and cooling. Each particular variety of glass requires its own treatment and much research has been necessary to obtain the data relating to each glass.

Final Examination. After annealing, the glass is repolished if necessary and examined upon the polariscope for strain. Annealing is not considered satisfactory unless the glass shows a uniform dull grey field under crossed nicols.

Optical Constants. Small cubes are cut from some of the fine annealed slabs and polished on two faces at right angles to one another. The refractive indices for the *C*, *D*, *F* and *G'* lines of the spectrum are then read on a Pulfrich type of refractometer. If the refractive index of a glass for the *D* line is higher than 1·7, its indices have to be read on a spectrometer. It is very seldom that the first melting of a new variety of glass gives within the third decimal place the refractive index required by the manufacturing optician, and often three or four attempts have to be made before the requisite glass is obtained. As a rule, once a glass of particular optical constants has been melted, further meltings can be made which will be "repeat" meltings from the standpoint of refractive indices to within ± 0·001.

Although optical glass is usually supplied by the manufacturer in slabs, polished at opposite ends, it is also sent out on occasions in other forms such as discs, lens blanks, and prisms. These are generally made in specially shaped moulds, and great care is necessary to ensure that no faults appear in the finished glass through moulding. At the Derby Works all discs, lens blanks, and prisms are made from glass previously examined for striae, whilst in slab form, so that the finished article can be guaranteed. This method is more expensive than pressing the disc or blank from a rough lump of optical glass, but is far more satisfactory because the rough lump can never be properly examined for fine striae. The price of discs and lens blanks prevents any polishing and examination for striae after manufacture, and thus a

method employing prior examination of the glass before moulding has much to commend it.

It has been found that the only sure method of testing for fine striae in glass is the examination of pieces of glass, polished on opposite faces, by means of a polariscope, and this is the method used at Derby.

3. THE SCIENTIFIC ASPECT OF MANUFACTURE

When Abbe and Schott developed their famous glasses they did so from arbitrary formulae and the refractive indices they obtained were put in possession of the optician. He in turn worked out his combinations and designed his machinery to suit these glasses. In the early days of the manufacture of optical glass at Derby the problem was the reverse of this and so much the harder. It was necessary to provide glasses of a particular refractive index to replace those originally obtained abroad, so that the optician need not alter his machinery nor have to recalculate his combinations. The first method employed by the author was to analyse specimens of existing types of glass, and from the composition so obtained make up and melt batch mixtures, in the hope of obtaining a glass like the specimen glass analysed. This method failed in its immediate object, for although optical glasses were obtained, they did not give the necessary optical constants so urgently required. It was decided therefore to abandon this method of analysis, which after all was only the crude method of the copyist, and could lead neither to the product of new types of glass nor to the improvement of the old ones. In its place the writer began a study of the influence of glass making oxides upon the optical and other properties of glass. This method of attack has proved most successful and the whole of the glasses now being produced at Derby have been worked out in this way. As far as possible the study is supplemented by analysis, but many glasses in pre-war use have been worked out without any of the original glass being available.

In the investigation of the influence of the composition of glass upon its properties, the following properties received most attention: (1) density, (2) refractive index, (3) durability, (4) tendency to devitrification. At the same time melting properties were explored and research was carried out upon the elimination of the various defects likely to be found in glass. Several new and important facts have been discovered regarding the properties of glass and it may be of interest to discuss briefly some of these discoveries. Fuller information as to the methods of research can be obtained from the papers of the author which have appeared in the *Journal of the Society of Glass Technology* (see appendix).

It was decided to work first of all with the simpler glasses and then investigate the more complex. The normal glass may be regarded as the result of the interaction of various chemical compounds to form new compounds, which at the high temperature of formation are mutually soluble and remain so when cold. The simplest normal glass is one consisting of two radicles, one basic and one acidic, and glasses of this type can be made by melting together silica and a compound of sodium or potassium. Thus melting together sand and sodium carbonate would give a glass

the analysis of which could be expressed in terms of silica and sodium oxide. The first glasses melted therefore consisted of silica and one of the alkali oxides.

The composition of a glass or the batch from which it is melted can be written down in two ways; either the oxide constituents may be expressed molecularly or in percentage weights. In most of the author's researches molecular proportions were used as a basis of calculation, the number of molecules of silica being expressed as 100 for convenience.

It is a well-known fact that although glasses made from sand and alkali oxides are useless from the point of view of durability the addition to them of a proportion of any of the oxides of calcium, barium, strontium, zinc, magnesium or lead results in a durable glass. Thus a study of glasses containing these oxides in addition to silica and the alkali oxide is of importance. As an aid to this study it was decided to take the two glasses 100 SiO_2 . 20 Na_2O and 100 SiO_2 . 20 K_2O and use them as nuclei upon which other glasses could be built. In addition, in order to obtain knowledge as to the action of the alkaline oxides in glass, the two glasses

100 SiO_2 . 40 Na_2O and 100 SiO_2 . 40 K_2O

were also used as "nuclei" glasses. It is not proposed to enter into a discussion of the results obtained from all the series of glasses made. It may be of interest to deal with some of the results, which indicate the specific actions of these oxides of calcium, barium, etc., in glasses containing 100 molecules silica and 20 molecules sodium oxide.

The scheme of melting for these glasses was as follows, the general molecular formula for the glasses being 100 SiO_2 . 20 Na_2O . xRO:

Glass No.	Molecules silica (SiO_2)	Molecules sodium oxide (Na_2O)	Molecules RO
1	100	20	5
2	100	20	10
3	100	20	15
4	100	20	20
5	100	20	25
6	100	20	30

The last column is headed "molecules RO" where RO represents the monoxide of one of the following metals: calcium, barium, strontium, zinc, magnesium, lead. Thus for example for the calcium oxide glasses a series of six meltings was carried out, each glass containing a constant molecular proportion of silica (100 mols.) and sodium oxide (20 mols.), whilst the proportion of calcium oxide increased from 5 molecules to 30 molecules, i.e. the only variant from a molecular standpoint was the calcium oxide. In the next series of meltings barium oxide was substituted for calcium oxide, and so on, lead oxide, strontium oxide, zinc oxide, and magnesium oxide being substituted in turn. The meltings as far as possible were carried out under similar conditions, and the properties of the glasses melted were investigated. From the results obtained the effect of increasing amounts of each oxide could be found, and also a comparison of the effects of the various oxides amongst themselves. The methods of melting and the modes of investigation of the various properties will not be dealt with here, as they are to be found in papers published by the author (see appendix).

A. Relationship between Composition and Density.

Effect of molecular addition.

Fig. 1 shows the effect of adding increasing molecular proportions of various oxides to the glass 100 SiO_2 . 20 Na_2O. The density of this glass is 2·35. The glass 100 SiO_2 . 20 Na_2O . 5 CaO has a density of 2·41, i.e. the addition of 5 molecules of calcium oxide to the "nucleus" 100 SiO_2 . 20 Na_2O increases the density 0·06. The glass 100 SiO_2 . 20 Na_2O . 5 PbO has a density 2·63. Thus the addition of 5 molecules of lead oxide to the "nucleus" increases the density 0·28. The following facts are noteworthy: (1) the addition of any of the RO oxides to the "nucleus"

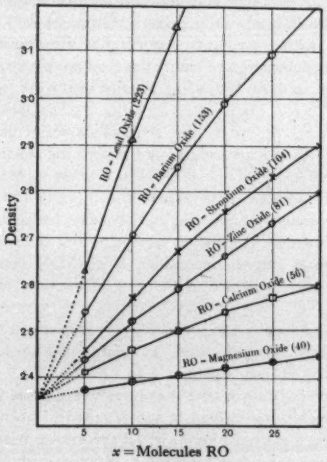


Fig. 1

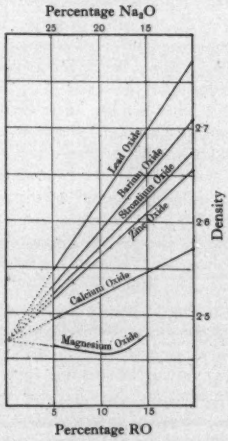


Fig. 2

Fig. 1. Molecular Composition and Density in the series 100 mols. SiO_2 . 20 Na_2O . xRO.
Fig. 2. Percentage Composition and Density (silica constant = 70 per cent.).

100 SiO_2 . 20 Na_2O results in an increase in density. (2) As the amount of any single oxide is increased, the density increases. Thus each curve is an ascending one. It will be seen, however, that successive equal additions to RO do not result in equal increases of density; the effect on the density value decreases as the molecular proportion of RO in the glass rises. (3) Of the oxides in question lead oxide has the greatest effect upon density when added molecularly to a glass, whilst magnesium oxide has least effect. A study of the following table shows that the greater the molecular weight of the oxide the greater is its effect upon density.

Oxide		Molecular weight	Density of 100 SiO_2 . 20 Na_2O . 10 RO
Lead monoxide	(PbO)	223	2·91
Barium oxide	(BaO)	153	2·71
Strontium oxide	(SrO)	104	2·57
Zinc oxide	(ZnO)	81	2·52
Calcium oxide	(CaO)	56	2·46
Magnesium oxide	(MgO)	40	2·39

Effect of percentage addition.

In the glasses investigated from the point of view of percentage composition by weight the glass 70 per cent. SiO_2 . 30 per cent. Na_2O was taken as the "nucleus" in the first series. The silica percentage was kept constant and an increasing percentage of sodium oxide substituted by RO.

Besides the series of glasses made with a constant proportion of silica equal to 70 per cent. other series were melted and investigated where the constant proportion of silica was 65, 60, 50 and 40 per cent. in turn. The same density laws were found to apply in all the series.

Fig. 2 shows the effect upon density in the 70 per cent. silica series of the substitution of all the RO types of oxides in increasing amounts, and the following facts are evident:

(1) In glasses of the type $a SiO_2 . b Na_2O . c RO$, where $a + b + c = 100$ per cent., as an increasing proportion by weight of the Na_2O is replaced by an oxide of the RO type, the silica remaining constant, there is an increasing rise in density, except in the case of glasses containing magnesium oxide. With MgO glasses, replacing Na_2O by MgO, a fall in density is caused until the glass contains about 10 per cent. MgO.

(2) In like glasses the greater the molecular weight of the RO type of oxide in the glass, the greater will be the density. Thus 70 SiO_2 . 20 Na_2O . 10 PbO has a density 2·61 and 70 SiO_2 . 20 Na_2O . 10 CaO has a density 2·52.

(3) It may be said that experiment has shown that the above two density laws apply when the alkali oxide is potassium oxide instead of sodium oxide.

(4) Another interesting fact may be mentioned here. If the percentage weights of silica and the RO type of oxide in a glass be kept constant, and the sodium oxide be replaced entirely or partially by potassium oxide, there is a fall in density, e.g.

Glass 70 SiO_2 . 20 Na_2O . 10 PbO	Density 2·610
,, 70 SiO_2 . 10 Na_2O . 10 K_2O . 10 PbO	,, 2·570
,, 70 SiO_2 . 20 K_2O . 10 PbO	,, 2·546

Four further series of glasses were studied where the percentage amount of sodium oxide was kept constant, and some of the silica replaced by RO. In the first series melted the Na_2O was kept constant at 25 per cent. and in the second series at 20 per cent. Two further series had constant amounts of Na_2O of 15 and 10 per cent. respectively. The same density laws were found to apply whatever the constant amount of Na_2O was in a series; they are as follows:

(1) In a series of glasses of the $a\mathrm{SiO_2} \cdot b\mathrm{Na_2O} \cdot c\mathrm{RO}$ type, where $a + b + c = 100$ per cent., if the percentage of $\mathrm{Na_2O}$ is kept constant and equal percentage weights of $\mathrm{SiO_2}$ replaced by RO, the density rises.

(2) In like glasses the greater the molecular weight of the RO type of oxide in the glass the greater will be the density.

It should be noted that the same laws apply if the sodium oxide is wholly or partially replaced by potassium oxide.

B. Relationship between Composition and Refractive Index.

Effect of molecular addition.

Small cubes were cut from each melt after annealing, polished on two adjacent surfaces, and the refractive indices for the *C*, *D* and *F* lines of the spectrum taken on a Pulfrich refractometer. Only the glasses dealt with in section A under Density

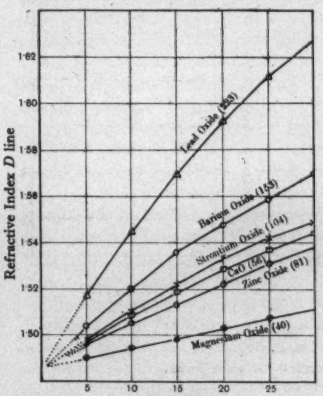


Fig. 3

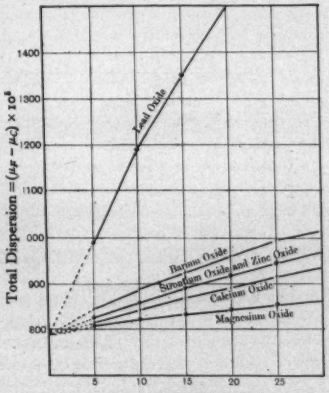


Fig. 4

Fig. 3. Molecular Composition and Refractive Index (μ_D) in the series 100 mols. $\mathrm{SiO_2}$. 20 $\mathrm{Na_2O}$. *x*RO.

Fig. 4. Molecular Composition and Total Dispersion in the series 100 mols. $\mathrm{SiO_2}$. 20 $\mathrm{Na_2O}$. *x*RO.

need be discussed here, so that the relationships between density and refractive index can be seen. Fig. 3 shows the effect upon the refractive index when increasing molecular amounts of the RO type of oxide are added to the glass nucleus 100 $\mathrm{SiO_2}$. 20 $\mathrm{Na_2O}$. The following observations have to be made:

(1) Molecular additions of any RO type of oxide to the glass nucleus

100 $\mathrm{SiO_2}$. 20 $\mathrm{Na_2O}$

causes an increase in μ_D, the greater the amount of xRO added the greater being the increase.

(2) In glasses of the type 100 SiO_2 . 20 Na_2O . xRO the greater the molecular weight of the RO type of oxide, the higher will be the value of μ_D, except in the case of zinc oxide. This oxide, although its molecular weight (82) is between the molecular weights of SrO (104) and CaO (56), has μ_D values lower than the corresponding calcium oxide glasses. As a matter of fact it is the calcium oxide glasses that are anomalous in their behaviour (see later).

Fig. 4 shows the total dispersion ($\mu_F - \mu_C$) of the same glasses as are shown in Fig. 3. It will be seen that molecular additions of any of the RO type of oxide occasion an increase in total dispersion, the greater the amount of RO added, the

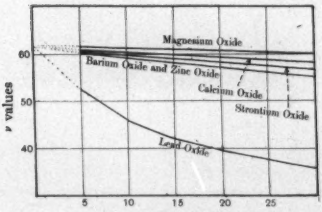


Fig. 5

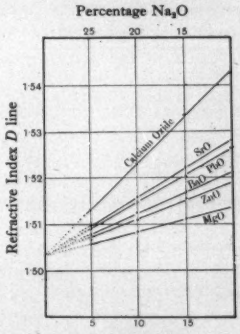


Fig. 6

Fig. 5. Molecular Composition and ν value in the series 100 mols. SiO_2 . 20 Na_2O . xRO.

Fig. 6. Percentage Composition and Refractive Index. (Silica constant = 70 per cent.)

greater being the increase. One striking inference to be drawn from the diagram is the large effect of PbO, when added molecularly, compared with the effect of the other oxides of the RO type. It will be noted that the values for ZnO are practically coincident with those of SrO, being slightly higher, although the molecular weight of ZnO is less than that of SrO. Fig. 5 shows the ν values of the glasses whose μ_D values are shown in Fig. 3. It will be seen that all the oxides except lead oxide have ν values close together, and much higher than lead oxide.

Effect of percentage addition.

Fig. 6 shows the effect upon μ_D when the percentage weight of silica is kept constant (= 70 per cent.) in a glass and some of the sodium oxide is replaced by oxides of the RO type. The following points are of interest:

(1) The values of μ_D rise as Na_2O is replaced by RO. Thus 1 per cent. of any of the RO oxides has more effect upon refractive index than the 1 per cent. of Na_2O it replaces.

(2) In like glasses (i.e. glasses containing the same percentage weight of SiO_2 . Na_2O and RO) the order of refractive index is *not* that of the molecular weight, as in the case of density. This is a very striking and unexpected result. Because flint glasses in optical use are so much higher in refractive index than crown glasses, it has become customary to regard lead oxide as being the oxide conferring high refractive index upon glass, particularly as compared with say calcium oxide. Fig. 6 indicates that *weight for weight both calcium oxide and strontium oxide have a greater effect upon refractive index than lead oxide, when they are substituted for sodium oxide in a glass.* In comparing the refractive indices of flint and crown glasses, the fact is often overlooked that the flint glass may contain 40 per cent. of lead oxide or more, whilst the crown glass seldom contains more than 15 per cent. of calcium oxide. Thus a false comparison is made. Where the percentages of CaO and PbO are equal in a glass, the calcium oxide glass will have a higher refractive index than the lead oxide glass, e.g. the glass 70 per cent. SiO_2 . 15 per cent. Na_2O . 15 per cent. CaO has a μ_D value 1·5320, whilst the corresponding glass 70 SiO_2 . 15 Na_2O . 15 PbO has a μ_D value 1·5204.

(3) Since lead oxide glasses of the type under discussion are higher in density than the corresponding calcium oxide glasses, it follows that a *rise* in density is accompanied by a *fall* in μ_D. A similar phenomenon is to be noted in comparing the MgO glasses amongst themselves. When MgO is substituted for Na_2O in a glass in which the SiO_2 remains constant, the density falls continuously until the glass contains 10 per cent. MgO, but the μ_D value rises continuously.

(4) The μ_D values of the calcium, strontium and barium oxides are in the reverse order of their molecular weights.

Fig. 7 shows the total dispersion of the glasses containing a constant amount of silica, namely 70 per cent. Again the order of total dispersion is not that of the molecular weights of the RO oxides, although lead oxide with the highest molecular weight has the highest total dispersion and MgO with the lowest molecular weight the lowest total dispersion. The calcium, strontium and barium oxides once more give anomalous results, the order of their total dispersion being the reverse of the molecular weight.

Perhaps the most striking feature of the total dispersion results is the fact that with strontium oxide, barium oxide, and magnesium oxide the total dispersion *falls* as the RO oxide increases. The effect of this is seen particularly in Fig. 8, which shows the ν value. In all cases except lead oxide, where RO is substituted for sodium oxide, the silica remaining constant, the ν value increases as μ_D increases, the effect being most marked with MgO, BaO, and SrO.

It should be said here that the above rules, worked out for glasses containing a constant amount of silica of 70 per cent., also apply to glasses containing constant amounts of silica of 65 and 60 per cent.

Fig. 9 shows the effect upon refractive index when the Na_2O is kept constant

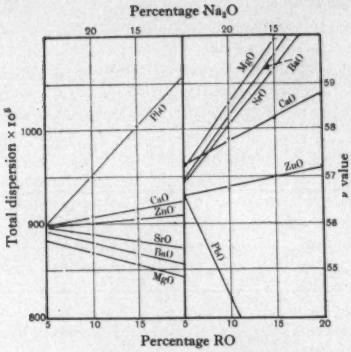


Fig. 7

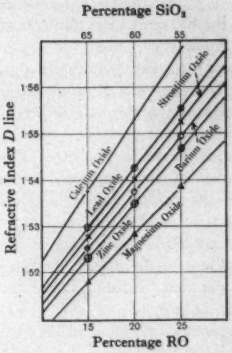


Fig. 8

Fig. 9

Fig. 7. Percentage Composition and Total Dispersion.
(Silica constant = 70 per cent.)

Fig. 8. Percentage Composition and ν value.
(Silica constant = 70 per cent.)

Fig. 9. Percentage Composition and Refractive Index.
(Sodium oxide constant = 20 per cent.)

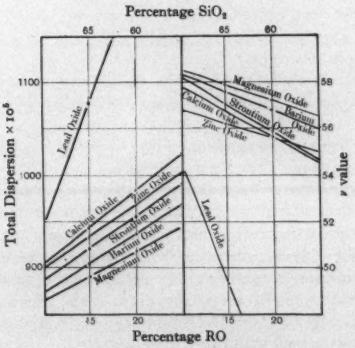


Fig. 10

Fig. 11

Fig. 10. Percentage Composition and Total Dispersion.
(Sodium oxide constant = 20 per cent.)

Fig. 11. Percentage Composition and ν value.
(Sodium oxide constant = 20 per cent.)

at 20 per cent. and silica is replaced by RO. Once more it will be seen that the index increases in every case, the rate of increase being greater than if Na_2O is replaced by RO (Fig. 6). Also the substitution of CaO has more effect than the substitution of PbO, whilst the effects of CaO, SrO, and BaO are in the reverse order of their molecular weights.

Fig. 10 shows the total dispersion of these glasses. In every case the total dispersion rises, so that the ν value falls (Fig. 11) as the μ_D value increases in these glasses.

Comparison of Figs. 6–11 proves, therefore, that in a glass of the

$$a\mathrm{SiO_2} \,.\, b\mathrm{Na_2O} \,.\, c\mathrm{RO}$$

type, if RO is substituted for SiO_2 the ν value falls as μ_D rises, whilst if RO replaces Na_2O the ν value *rises* as μ_D rises, except in the case of lead oxide.

Before concluding this section upon refractive index, it should be said that, although the above laws have been deduced from sodium oxide glasses, they apply equally well to glasses containing potassium oxide. It should be stated also that the substitution of potassium oxide for sodium oxide in a glass, weight for weight, results in a fall in refractive index and in total dispersion.

The necessary data having been obtained for glasses of the type silica-alkali-RO, it remained for glasses of a more complex kind to be dealt with. A very important research was upon the effect of boric acid in various glasses. Then the part played in glass by the R_2O_3 type of oxide, such as aluminium oxide, antimony oxide and arsenious oxide, was tested. Afterwards, when dealing with the barium flint glasses, the effect of one RO type of oxide in the presence of another RO oxide was investigated. Lack of space forbids any discussion of the results obtained, but it will be seen that sufficient data regarding refractive indices were available to allow batch mixtures to be worked out for a large number of glasses. As a rule, in the search for a certain glass of specific refractive indices, several batches could be worked out, and the problem then was one of deciding the best batch to use.

Having determined the question of refractive indices, the choice of a batch is mainly dependent upon three further factors: (1) melting properties, (2) durability, (3) tendency to devitrification. Generally considerable information regarding melting properties was gleaned from the small meltings, but further research had to be carried out upon durability and devitrification. Each glass therefore was subjected to tests of durability and devitrification.

C. Relationship between Composition and Durability.

It is demanded of good optical glass that, after polishing, it shall keep its bright surface without tarnishing or disintegration over a long period. Different polished glass surfaces are affected in various ways. Some are covered with blotched, circular stains; others may become greasy. Again, glasses containing much barium or lead often become covered with iridescent-coloured patches. Some glass surfaces will scale, and become even entirely disintegrated. The effect upon a polished glass surface on exposure is termed "weathering."

The durability of a glass, or its weathering properties, can be tested in various ways. The best and only sure method is the actual effect on the glass lens or prism after exposure for a considerable time in an instrument under working conditions. Before a glass can be introduced into optical practice, tests which can be carried out quickly must be applied, and these tests may be many and various. Exposure to steam under pressure, to acids and alkalis, to hot water, to various artificial atmospheric conditions, all find a place in testing, and the results obtained, as a rule, are parallel with those of the longer and surer method of actual use*. In conjunction with the results gained by the opticians for glass under service conditions, these laboratory tests are of considerable value. The author has carried out a large number of experiments on optical glasses, both in actual use and in course of development, with a view to discover, firstly, how far the laboratory tests are in line with one another and with the actual behaviour in every day use, and, secondly, which is the simplest and most expeditious laboratory test yielding results of value.

For general purposes the solubility of powdered glass in water has been found best. Like all laboratory tests for judging processes it is open to objection, but in skilled hands it has yielded valuable results. Briefly the test is as follows: The glass is powdered, dried, and passed through a 160 hole sieve. Five grams of the powder are digested with 100 cc. of distilled water in a platinum basin for 1 hour at 80° C. The undissolved matter is filtered off, and the amount of acid required to neutralise the filtrate obtained by titration. The results are generally expressed in milligrams of alkali dissolved from 100 grams of glass. It has been found that a glass requiring more than 1000 milligrams of sulphuric acid to neutralise the filtrate per 100 grams powdered glass after the above treatment cannot be regarded as useful for optical glass purposes. To be of really first class quality for weathering less than 200 milligrams should be required. Many of the glasses made at Derby conform to this standard; few require more than 400 milligrams, and these are required for special purposes†.

No attempt can be made to give a full account of all the results obtained, but a short reference to some of the glasses dealt with previously in this paper may be of interest in showing the effect of the various RO types of oxide upon the solubility of glass in water (or more correctly upon the degree to which the more soluble constituents can be extracted).

Molecular composition and solubility.

Fig. 12 indicates the solubility of glasses of the molecular series

$$100\ SiO_2 . 20\ Na_2O . x RO,$$

the results being expressed in milligrams of sodium oxide extracted from 100 grams of glass. The following remarks are to be made:

* A discussion of the various tests of the durability of glass, together with a very full bibliography on the subject, will be found in *Journ. Soc. Glass Techn.* 1 (1917), 160–161 ("The Attack of Chemical Reagents on Glass Surfaces and a Comparison of Different types of Chemical Glassware." By Cauwood, English and Turner).

† See *Journ. Soc. Glass Techn.* vol. 4, p. 39.

(1) The addition of any oxide of the RO type to the nucleus glass 100 SiO_2 . 20 Na_2O results in a fall in solubility. The effect is very pronounced at first. Thus 75 per cent. of the nucleus glass 100 SiO_2 . 20 Na_2O will dissolve in water under the conditions of test. The addition of 5 mols. of calcium oxide reduces this solubility enormously, only $2\frac{1}{2}$ per cent. of the glass 100 SiO_2 . 20 Na_2O . 5 CaO being soluble.

(2) Increasing molecular additions of RO decreases the solubility, but the rate of decrease falls away. This is well illustrated by Fig. 12.

(3) The order of effect upon solubility of glass of the RO oxides added molecularly is not in the order of their molecular weights. A remarkable feature is the large effect of zinc oxide.

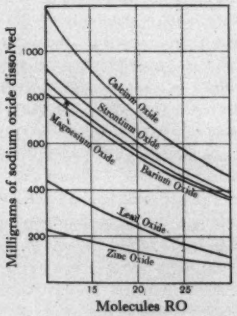


Fig. 12

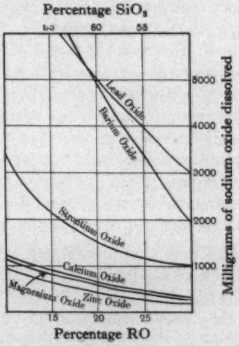


Fig. 13

Fig. 12. Molecular Composition and Solubility of Glasses.
100 SiO_2 . 20 Na_2O . xRO.

Fig. 13. Percentage Composition and Solubility.
Sodium oxide constant = 20 per cent.

Percentage composition and solubility.

Fig. 13 shows the effect upon solubility of replacing the silica by the RO type of oxide, the sodium oxide being kept constant (20 per cent.). It will be seen that in every case there is a decrease in solubility and the order of the effect is in the order of the molecular weights of the RO oxides, with the exception of zinc oxide. Thus the *greater* the molecular weight of the oxide added the *less* will be the effect upon the lowering of solubility, except in the case of zinc oxide which has the greatest effect of all. This latter fact is of great importance. In another series of results obtained, where the silica was kept constant and the Na_2O replaced by RO, it was again found that the solubility was reduced, and the diagram obtained by plotting these results (not shown) is very similar to Fig. 13, except that the curves

are steeper. In this case also the curves for ZnO, MgO, and CaO are almost coincident.

In order to test the efficacy of using the solubility test as a criterion of weathering, polished pieces of each of the glasses made were subjected to actual weathering tests. In the first place they were exposed for 100 hours to air saturated with moisture at 18° C. and secondly they were exposed for six months to ordinary atmospheric conditions. In every case the order of weathering, i.e. the order of durability, was identical with the order of solubility.

It should be stated here that if sodium oxide is replaced by potassium oxide in these glasses of the $a\mathrm{SiO_2} \cdot b\mathrm{R_2O} \cdot c\mathrm{RO}$ type, where $a + b + c = 100$ per cent., the solubility and therefore the durability is altered. Where the alkali content is greater than 15 per cent., the $\mathrm{Na_2O}$ glass usually has the better durability. Below 15 per cent. alkali the $\mathrm{K_2O}$ glass is usually the more durable, although no definite rule can be stated. This matter however is of less importance than at first sight appears, because of a somewhat remarkable generalisation discovered by the author which can be stated as follows: *in the* $\mathrm{SiO_2}$*-alkali-*RO *glasses, when the total alkali is less than* 20 *per cent., the glasses containing both alkalis in equal proportion by weight will be less soluble and more durable than the glass containing all its alkali as potassium oxide or as sodium oxide. This is true whatever the percentage of* $\mathrm{SiO_2}$ *or* $\mathrm{R_2O}$ *in the glass.*

Thus the glass 70 $\mathrm{SiO_2}$. 5 $\mathrm{Na_2O}$. 5 $\mathrm{K_2O}$. 20 PbO will be less soluble and more durable than either of the glasses

70 $\mathrm{SiO_2}$. 10 $\mathrm{Na_2O}$. 20 PbO or 70 $\mathrm{SiO_2}$. 10 $\mathrm{K_2O}$. 20 PbO.

Experiments have been conducted to find the most suitable ratio between $\mathrm{Na_2O}$ and $\mathrm{K_2O}$ to give maximum durability (see *Journ. Soc. Glass Techn.* vol. 5, p. 195). The following rules can be deduced from the data given above regarding the durability of glasses of the type $a\mathrm{SiO_2} \cdot b\mathrm{R_2O} \cdot c\mathrm{RO}$, where $a + b + c = 100$ per cent.:

(1) Durability can be improved by replacing some of the silica by RO, the alkali remaining constant.

(2) Durability can be improved by keeping the silica constant and replacing some of the alkali by RO.

(3) In the improvement of durability by replacing either $\mathrm{SiO_2}$ or alkali weight for weight by RO the best oxide to use is ZnO. The other RO oxides improve durability in the order of their molecular weights, MgO with lowest molecular weight having the greatest effect upon improvement, PbO the least.

(4) The durability of glass is improved by the presence of both alkaline oxides $\mathrm{Na_2O}$ and $\mathrm{K_2O}$.

D. Relationship between Composition and Devitrification.

As previously stated glass may be regarded as the result of the interaction of various chemical compounds to form new compounds, which at the high temperature of formation are mutually soluble and remain so when cold. Under certain circumstances some of the compounds mutually soluble at high temperature may separate out when the melt cools, forming opaque masses of varying size, shape, and composition. This separation under these conditions is termed "devitrifica-

tion" and has to be guarded against in the manufacture of optical glass. It is impossible to go deeply into the phenomenon here, but it is discussed at great length in various papers published by the author (see appendix). The following facts are of interest:

Silica separation. In practically all glasses, with the exception of some of the dense barium crowns and very dense flints, the predominant oxide from a percentage point of view is SiO_2. Accordingly the separation of silica from molten glass is perhaps the commonest type of devitrification. This separation of silica is governed by several factors. In the first place, the rate of cooling may determine whether silica will separate. Thus a glass 70 per cent. SiO_2 . 15 Na_2O . 15 PbO, on cooling rapidly 100° C. per hour, showed no signs of devitrification. When cooled more slowly, viz. 50° C. per hour, silica separated out. Secondly, the alkali oxide present may determine the separation of SiO_2. For example, the glass 70 per cent. SiO_2 . 15 Na_2O . 15 BaO separates silica, on cooling at the rate of 50° C. per hour, whilst the glass 70 per cent. SiO_2 . 15 K_2O . 15 BaO does not separate silica when cooled at the same rate. The reason is probably a matter of viscosity. Sodium oxide is, weight for weight, a better flux than potassium oxide and for a given temperature a sodium glass is more fluid than its corresponding potassium glass. The less viscous a glass is, the more easily can crystallisation nuclei form and develop. A third factor governing the separation of silica is the type of RO oxide present. Thus silica separates more easily from a glass containing lead oxide than from one containing the same percentage amount of calcium oxide. It is almost impossible to make by optical processes flint glasses (i.e. glasses containing lead oxide) when the silica percentage is greater than 65, yet crown glasses (i.e. glasses containing calcium oxide) can be made with the silica percentage as high as 75 without separation of silica on slow cooling.

When silica separates from a melting it may do so in the form of white opaque "blobs" or in the form of crystals. Photograph 4 shows crystalline and opaque separation of silica. The glass from which these separations took place contained 62 per cent. silica and 25 per cent. lead oxide. Photograph 5 shows one of the hexagonal crystals more highly magnified and Photograph 6 a triangular silica separation found in this melt. It is of note that the crystals shown separated out along a stria in the glass, the bulk of the melt being quite clear. They were somewhat smaller than a pin head in actual size. Occasionally the silica separates in such a manner as to cause the whole melting to appear opalescent. Thus a melting containing 35 per cent. PbO and 54 per cent. SiO_2, on slower cooling than usual, was found to be milky throughout. Again a glass on first cooling in the furnace may be quite clear, but on being subjected to a second heating and cooling during the moulding process may separate silica and appear opalescent. Such a glass is a light barium crown containing 61 per cent. SiO_2, 19 per cent. BaO. As a rule when silica separates from glass in a crystalline form it does so in the form of tridymite (Photographs 5 and 6). Only on one occasion and by a special method has cristobalite been crystallised from glass by the author. This is shown in Photograph 7.

Separation of silicates of "RO" metals.

The amount of the RO type of oxide in a glass often determines the tendency of a glass to devitrification. For example, the glass 65 per cent. SiO_2 . 15 Na_2O . 20 CaO on slow cooling separates calcium silicate. As a rule, when the CaO content of a glass is higher than 15 per cent., there is a tendency for calcium silicate to separate out. This separation takes the form of fine needle-like crystals as shown in Photograph 8. Often these needles radiate from a common centre and form a perfect sphere in the glass. Such spheres have been known as large as cricket balls. With barium glasses the tendency to separate barium silicate on slow cooling is not marked until a content of 25 per cent. BaO is reached. Then needle-like crystals of barium silicate separate. Photograph 9 shows such a separation. Dense barium crown glasses which often contain 40 per cent. of BaO, are especially prone to devitrification, the separation taking place along striae as a rule. Usually the form of crystal shown in Photograph 9 is met with, but occasionally, especially after a second heating and cooling, another type of barium silicate separation is known. Photographs 10 and 11 show this type of formation. Both these photographs are of exactly the same piece of glass, the beautiful "flower garden" effect in No. 10 being obtained with crossed nicols.

Glasses containing lead oxide are unique from the fact that more than 50 per cent. PbO can be introduced without causing devitrification. In fact it is almost impossible to induce devitrification in glasses containing a high percentage of lead oxide, and only one such devitrification has been achieved by the author. Photograph 12 shows lead silicate crystals which were prepared by slowly cooling a glass containing about 80 per cent. lead oxide.

4. CONCLUSION

No mention has been made of the investigations upon colour in optical glass, upon the elimination of bubble, upon refractories, upon annealing, etc., all of which require continuous attention. One point regarding colour may be made here. It has been found without exception that glasses containing potassium oxide are always less coloured and more brilliant than corresponding glasses containing equal amounts of sodium oxide. This may be due to the fact that potassium glasses exert a less corrosive action upon pot clay than the sodium glasses and so absorb less ferruginous matter, a cause of colour in glass. A more likely reason is that in glass potassium compounds containing iron are less coloured than the corresponding sodium compounds.

Without further reference to these matters, however, sufficient evidence has been produced to show that the whole manufacture of optical glass is practically research upon a manufacturing scale, accompanied by all the attendant expenses of research, by its disappointments and failures as well as by its successes. Perhaps in no other manufacture have these disappointments and failures to be faced so many times before success is reached. It will be seen therefore that only with the sympathetic support and encouragement of the users of optical glass can its manufacture continue.

The author is indebted to Mr W. Morritt for taking the photographs shown in this paper.

APPENDIX

Modern papers upon Optical Glass Production.

A. *Papers by the author.*

"The Development of Various Types of Glass," etc. (Peddle). *Journ. Soc. Glass. Techn.* 1 (1917), 27–61; 4 (1920), 3–107, 225–238, 281–283, 299–366; 5 (1921), 3–15, 72–106, 195–267.

B. *General Papers.*

1. "Formation of Seed in Optical Glass Melts" (Williams). *Journ. Amer. Ceramic Soc.* 1 (1918), 134.
2. "Milkiness in Optical Glass" (Fenner and Ferguson). *Ibid.* 1 (1918), 468, 561.
3. "Milkiness in Optical Glass" (Gelstharp). *Ibid.* 1 (1918), 559.
4. "Technique of Optical Glass Melting" (Fenner). *Ibid.* 2 (1919), 102.
5. "Optical Glass Manufacture" (Morey). *Ibid.* 2 (1919), 146.
6. "Devitrification of Glass" (Bowen). *Ibid.* 2 (1919), 261.
7. "Optical Glass Manufacture" (Williams and Rand). *Ibid.* 2 (1919), 422.
8. "Devitrification of Glass" (Cox). *Ibid.* 2 (1919), 576.
9. "Types of Optical Glass" (Montgomery). *Ibid.* 3 (1920), 404.
10. "Rolled Optical Glass" (Roberts and Hostetter). *Ibid.* 3 (1920), 750.
11. "Composition and Refractivity of Optical Glass" (Wright). *Ibid.* 3 (1920), 783.
12. "Composition of Lead Glasses" (Montgomery). *Ibid.* 3 (1920), 900.
13. "Striae in Optical Glass" (Smith, Merritt and Bennett). *Bureau of Standards*, paper 373.
14. "Striae in Optical Glass" (Michelson). *Ibid.* paper 333 (1919).
15. "Production of Optical Glass" (French). *Optician*, vol. 59, pp. 59, 335, 351; vol. 60, pp. 3, 18, 28.
16. "Viscosity of Optical Glass" (Washburn and Shelton). *Phys. Rev.* 15 (1920), 149.
17. "Manufacture of Glass Melting Pots" (Brownlee and Gorton). *Journ. Amer. Ceramic Soc.* 4 (1921), 97.
18. "Electrical Annealing Lehrs" (Collins). *Ibid.* 4 (1921), 335.
19. "Composition of Barium Glasses" (Montgomery). *Ibid.* 4 (1921), 536.

C. *Papers on Furnaces.*

1. *Journ. Amer. Ceramic Soc.* 2 (1919), 114 (Fenner). See B. 4 above.
2. *Ibid.* 2 (1919), 426 (Williams and Rand). See B. 7 above.
3. *Optician*, vols. 59 and 60 (French). See B. 15 above.

D. *Papers on Stirring Machines.*

1. *Journ. Amer. Ceramic Soc.* 2 (1919), 129 (Fenner). See B. 4 above.
2. *Ibid.* 2 (1919), 426 (Williams and Rand). See B. 7 above.
3. "The Viscosity of Glass" (Washburn). *Journ. Amer. Ceramic Soc.* 3 (1920), 735.

E. *Papers on Annealing.*

1. "The Annealing of Glass" (Twyman). *Journ. Soc. Glass Techn.* 1 (1917), 61–74.
2. "The Cooling of Optical Glass Melts" (Roberts). *Journ. Amer. Ceramic Soc.* 2 (1919), 543.
3. "Annealing Temperatures of Optical Glass" (Littleton and Roberts). *Ibid.* 4 (1921), 224.
4. "Annealing of Optical Glass" (Adams and Williamson). *Ibid.* 4 (1921), 213.
5. "Annealing of Optical Glass" (Tool and Valesek). *Bureau of Standards*, paper 358 (1920).

DISCUSSION

Dr M. W. Travers thought that very few people realised the great amount of valuable work which the author and the Derby Crown Glass Co., had done during the war. He himself had found Dr Peddle's results of considerable value even in the case of bottle manufacture.

Dr J. W. French said he realised how small was the amount of published literature on the subject of optical glass manufacture. Before the war his firm had found it necessary to anneal electrically the optical glass received by them and shortly after the outbreak of war it became necessary to manufacture optical glass for rangefinder requirements, as for such work large pieces of glass of an exceptionally high quality are essential. No assistance of a really practical kind was obtainable from any published literature and to future investigators Dr Peddle's paper would appear as an oasis of excellent information in a somewhat arid desert.

In the historical portion Dr Peddle's reference to the work of Harcourt was slight. It was not generally realised how much Harcourt had done. Much that was attributed to other workers had been very fully investigated by him. It was unfortunate that he had published only one report dealing with his high pressure hydrogen furnace and that it had been left to Stokes to record in a very fragmentary way some of the important results achieved by this pioneer worker.

Faraday also had contributed greatly, and possibly in a more practical manner, to the development of optical glass manufacture, although his method is not the one that has been developed. It is interesting that both Harcourt and Faraday commenced by investigating vitreous substances containing three and possibly four ingredients and later concentrated upon the development of one or more particular types of glass. Dr Peddle has followed in the footsteps of these early workers. He thought that when Dr Peddle reached the stage of combining four or five or six different ingredients, as is necessary in practical work, he would concentrate as these earlier workers did on particular types.

Dr Peddle had omitted from his list of optical glass desiderata the quality of homogeneity, which really should be placed first on the list. Strain in good optical glass is a malady that can frequently be remedied by re-annealing, but want of homogeneity is an incurable disease. The omission, he thought, was probably accidental, but he would be glad to have this confirmed, as during the war the opinion had been expressed in certain quarters that want of homogeneity was not of first importance.

In the diagram of silica, alkali, and calcium types, the curve was carried as far as 20 per cent. of calcium, although the last indication on the calcium content line was 15 per cent. He would be glad to know if a specimen of this type containing 20 per cent. of calcium had actually been made. In investigating the same series he had found it necessary to use the oxy-acetylene furnace for the fusion of this hard glass.

In dealing with the subject of weathering, he was interested to note that, in the heavy lead glass series, good results were obtained by the addition of particular

proportions of soda and potash. Faraday in the production of heavy lead glasses emphasises the importance of excluding all alkali, as lead glasses containing a small proportion of alkali are affected by weathering, his explanation being that the alkali absorbed moisture which was retained as a film on the surface of the glasses. This film held in solution the sulphurous gases of the air which then re-acted on the lead.

It is interesting that these earlier workers generally called their products "vitreous substances," not necessarily "optical glass." Of the many melts illustrated by Dr Peddle it is remarkable what a small proportion could really be called "optical glass." Take, for example, the simple crown glasses, in which the percentage of calcium is varied. At the low calcium end the glasses absorb moisture. They are deliquescent vitreous substances. Over a small portion of the central region the glasses are good optical glasses. Over the higher regions they are either too hard to be stirred under present day conditions or they become opalescent or devitrify owing to the excess of some constituent. This condition of affairs is much more marked in many of the other series.

There is one type of surface devitrification which Dr Peddle had not referred to or exhibited. When certain glasses are heated, one of the constituents may be volatilised and leave the surface in a crystalline condition, the expansion of the crystalline surface being such that the glass may be very highly stressed or shattered or even split into layers. He (the speaker) had found that this difficulty could be overcome by enclosing the plates during the annealing process and introducing some of the material which volatilised, thus increasing the vapour pressure of this particular material and preventing loss from the surface of the glass.

He appreciated very much the great scientific service that Dr Peddle had rendered in so freely publishing the results of his labours. Although similar work had been done by the earlier pioneers and by optical glass manufacturers of more recent times, they had been unable to publish, or in some cases had refrained from publishing, their results. It was to be hoped that Dr Peddle's very excellent example would be more freely followed by others in future.

Professor Cheshire said that he had given the author a number of questions as to the determination of the refractive indices of any given melt, and he had kindly consented to give the requisite information.

Mr E. A. Coad-Pryor asked if the author had noticed in the case of other oxides any behaviour similar to the effect of mixing potash and soda.

Dr J. S. Anderson suggested that in testing a specimen of glass for striae or strain it is not necessary to polish optically flat faces. The necessary information can be obtained by immersing the specimen in a liquid having the same refractive index for some wave length within the visible spectrum. When the liquid has been adjusted roughly to give equality of index for some given colour, an accurate balance can be obtained very easily by varying the wave length of the light with which the specimen is being examined (see *Trans. Opt. Soc.* 21 (1919–20), 195). In this way it is possible to examine specimens having ground surfaces or irregular shapes.

Lt.-Col. Gifford asked if the author had tried the effect of thallium oxide.

He said that in the design of one-radius eyepieces it was necessary to obtain

flint glasses having a low refractive index and a dispersion twice, three times, or four times that of the crown components.

Mr P. F. Everitt said that with moulded lenses the cost of grinding away the rough outer layers was greater than the cost of the glass wasted. He would like to know what was the minimum amount of glass that had to be ground away in an average batch of specimens.

Mr T. Smith asked how the transparencies of glasses whose indices had been raised to the same extent by the introduction of calcium on the one hand and of lead on the other would compare with one another. It would be interesting, if the author had made such experiments, to see the influence in his experience exerted by the various oxides for glasses having the same ν value.

He wished to suggest that some of the results exhibited would be of particular interest to the optician if they were shown on a diagram with the refractive index as abscissa and the ν value as ordinate, the series formed by the addition of the various oxides then taking the form of a group of lines radiating from the point which represents the original soluble glass. The influence of the added oxide on the properties in which the optician is most interested would be rendered apparent in this way while divested of matters with which he is not primarily concerned. If the portion of the line which corresponds to the stable range of glasses could in these diagrams be differentiated from the parts which do not yield an optical glass the information conveyed by these diagrams would be of still greater value.

Author's reply (partly communicated):

Dr French raised the question of homogeneity. It could not be said that this important matter had been omitted, as the presence of striae in glass, and the stringent manner of testing for striae at Derby by the polariscope, had been emphasised. In a general paper it was impossible to dwell specifically on any one point, but the whole matter of homogeneity is one to which a whole evening's discussion might be devoted.

The specific glass enquired for by Dr French containing 20 per cent. calcium oxide, 70 per cent. silica, and 10 per cent. sodium oxide, had been made on a small scale, and the various points on the curve could be plotted from the data obtained. Its μ_D value is 1·542. It is a matter of interest that this glass cannot be made as an optical glass, as it would devitrify on slow cooling. It is possible to introduce as much as 40 per cent. calcium oxide into a melt, and by very rapid chilling obtain transparent glass. The extra amount of lime in these glasses replaces silica.

He agreed with Dr French that in the crown glasses containing calcium and alkali there was only a very small proportion that gave optical glass. More than 72 per cent. of silica gives separation of silica on slow cooling and, with more than 15 per cent. of lime, calcium silicate was likely to separate out. If more than 10 per cent. alkali was introduced the glass was bad from the point of view of durability.

Faraday's statement regarding the good weathering properties of very heavy lead glasses when they contain the minimum amount of alkali probably referred to the iridescing of the polished surface. Two kinds of weathering have to be

recognised. In the first place there is weathering that takes place in pure air, the definition of pure air being oxygen, nitrogen, inert gases and water vapour. In this kind of weathering the water vapour attacks the alkali silicates in the glass, resulting very probably in a thin film of alkaline hydroxide. In its worse form this kind of weathering can be recognised by the greasiness of the polished surface, or even by spots of liquid appearing on that surface. A second form of weathering is that due to attack on the glass surface by impurities in the atmosphere such as carbon dioxide, vapours of mineral acids, sulphur dioxide, and hydrogen sulphide. This carbon dioxide, sulphur dioxide and hydrochloric acid vapour in the presence of moisture may attack the glass surface, giving a whitish film. Hydrogen sulphide as a rule causes a glass surface to iridesce, particularly if the glass contains lead. From experience it has been shown that a polished piece of very dense flint glass sealed up in perfectly dry hydrogen sulphide vapour preserves its polish. If moisture is present the glass immediately blackens. Heavy lead glasses containing as much as 10 per cent. alkali are very resistent to the attack of moisture, or even acid vapours, but they rapidly iridesce in the presence of hydrogen sulphide.

He did not agree with Dr French that it would be practically impossible to investigate the effect of various oxides in glass containing four or more constituents. As a matter of fact a considerable amount of work had been done upon the effects of boric acid on the glasses discussed, also upon the effect of barium oxide in the presence of the oxides of lead and zinc.

Dr French's remarks on surface devitrification were most interesting and instructive. With the majority of glasses made at Derby, the amount of surface devitrification during annealing was extremely small.

Prof. Cheshire asked for information upon the following points:

(1) How many pieces of glass were taken from a melt in order to find the refractive index?

(2) From what portion of the melt were these pieces taken, and how did they differ one from another in index?

(3) How far was it possible to repeat the indices of a glass upon second meltings of the same type?

In the early days of manufacture, it was our custom to cut six pieces from various parts of the melt, care being taken to choose as far as possible homogeneous glass. It was found that these pieces were identical in index as far as the fourth decimal place, as read on the Pulfrich refractometer.

At present our practice is not to concentrate on the glass as taken from the pot, but to be most particular about the index of the finished fine annealed glass. It has been found by experiment that refractive index alters with the state of annealing. This can be well illustrated as follows. A piece of molten glass was taken from a dense barium crown melt just after stirring and allowed to cool down rapidly. Its index was found to be 1·6068. When the pot was broken up the index of the material was found to be 1·6097. After the moulding process the index of the glass was found to be 1·6083 and after the fine annealing process the index was 1·6112. Pieces of the fine annealed glass were then pressed into lenses and rough annealed.

It was found that their indices varied between 1·6088 and 1·6092. Each piece of glass was then again put through the fine annealing process and it was found that the index of all had become 1·6112. Thus it is the index of the fine annealed glass that really matters.

The accuracy with which successive meltings of the same type of glass can be repeated varies with the type of glass. Thus 15 successive meltings of a certain light flint glass did not vary more than 0·0004 from the mean index for the *D* line. On the other hand in a series of successive meltings of dense barium crown the discrepancy from the mean value was 0·0009. The failure to repeat index more exactly is a matter somewhat beyond our control, as it is due to the attack of the molten glass upon the fireclay of the pot. The greater the corrosion the greater will be the silica and alumina absorbed, and the greater will be the discrepancy in index. Dense barium crown glasses, having a greater corrosion action than light flint glasses, show greater discrepancies in index on repeat meltings.

In answer to Mr Coad-Pryor, attempts had been made to determine whether a mixture of two oxides other than those of potash and soda had the effect of improving durability. It is a very difficult matter to determine owing to pot attack upsetting the results, because the order of weathering is so much less than in the case of potash and soda.

Dr Anderson's suggestion that rough specimens of glass could be tested for striae or strain by the immersion method is of course a very valuable one, and has been employed by various people for testing small rough lenses and other small pieces of glass. It is, however, hardly applicable on the works scale because of the large amount of different solutions that would have to be kept, many of them costly and some of them easily decomposed.

In answer to Col. Gifford, no melts using thallium oxide had been made at Derby, but it was very probable that this oxide could be used to make glasses of a higher density even than the lead glasses, although of course the expensive nature of these glasses would tell against their production.

Mr Everitt's question was a very pertinent one, and from the point of view of the manufacturing optician perhaps the most useful question of the evening. Work was taking place now at Derby to try and mould lenses and prisms with such accuracy that there would be the minimum amount of grinding away necessary. It was hoped that an accuracy could be obtained such that in the smaller lenses up to 2 ins. diameter not more than $\frac{1}{2}$ mm. would have to be removed in grinding to the finished size.

He had no data regarding the transparencies of such calcium and lead glasses as Mr Smith had mentioned.

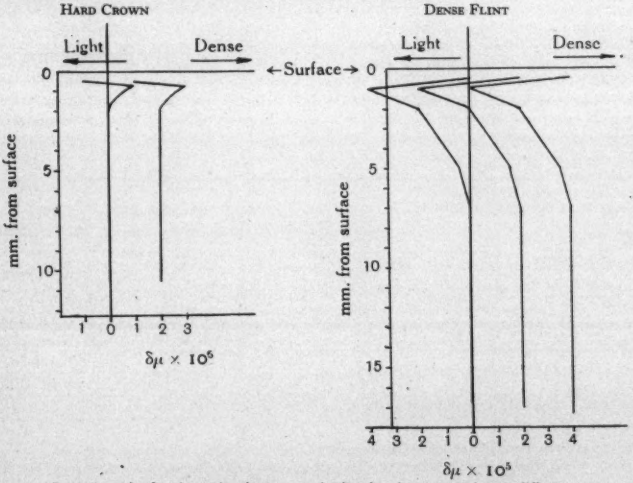
With regard to the plotting of results the method employed by the author is the one he has found most useful in the work he had in hand. It is quite possible from the data given to plot refractive index against the ν value as suggested by Mr Smith.

VARIATION IN REFRACTIVE INDEX NEAR THE SURFACES OF GLASS MELTS

By F. TWYMAN, F.Inst.P. and A. J. DALLADAY, A.Inst.P.

MS. received, 21st January, 1922. Read, 9th February, 1922.

Two pieces of glass from the top of each melt, one of dense flint, the other hard crown, were polished on two parallel faces normal to the surface to a thickness of 13 mm., and examined on the interferometer. The surface which in the crucible was exposed to the air was untouched. The interference patterns are shown graphically, the variation of refractive index indicated by bands of mercury green light being plotted against millimetres from the surface.



Deviation of refractive index from normal. One band corresponds to a difference of refractive index of 2×10^{-5}.

This note is communicated by the permission of the Director of the British Scientific Instrument Research Association, for which Association the observations were conducted.

THE OPTICAL THREE APERTURES PROBLEM

By T. SMITH, M.A., F.Inst.P.
(Optics Department, National Physical Laboratory)

MS. received, 30th November, 1921. Read, 12th January, 1922

SUMMARY. In such an instrument as a submarine periscope where broad beams of light have to be transmitted down a long tube from a wide field the relation between the length and diameter of the tube and the number of lenses is considered. In the case adopted as a standard of reference there are three effective apertures, corresponding to the visible field, the effective aperture of the objective and a third at the intersection of the two cones which have their vertices respectively at the centres of these apertures and pass through the rim of the remaining aperture. The light intensity then falls off continuously from the centre to less than half its original value at the edge. The length of body available is directly proportional in this case to the number of lenses, and any number of lenses may be chosen. Certain aberrations will have a minimum value when there are three lenses to every inversion, the reduction of this number by the omission of a lens being equivalent to an addition for aberration purposes. Advantages may be gained as regards light at the cost of greater aberrations; for instance if only field lenses and objectives situated at images of the original objective are used the illumination will be uniform all over the field. In a long instrument the price to be paid for this advantage—doubling the amount of the secondary spectrum and the curvature of field—is too heavy, and the construction generally used in recent years is preferable. Alternative types of construction which offer some advantages are indicated. The method employed is applicable to other instruments.

In examining an extended object with the aid of an optical instrument the useful rays of light are obviously limited by two apertures or stops, the rim of the pupil of the eye (unless it is not filled) being one and the boundary to the field of view the other. In some cases these are the only effective apertures. The eye then receives full pencils of light from all parts of the field as shown in Fig. 1. More often the field of view is enlarged by the inclusion of an outer zone the pencils from which are partially eclipsed by an intermediate aperture, for instance by the

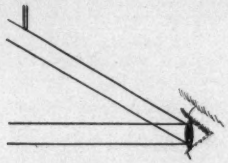


Fig. 1

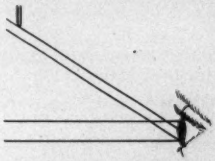


Fig. 2

rim of the mounting of the eye lens of the instrument, as illustrated in Fig. 2. In this case there are three effective apertures; more complex cases can almost invariably be reduced to this one. In general if the sizes and positions of the apertures are known the main properties of the instrument, such as the extent of the field and its illumination, may be derived. When a new instrument is required the information given to the designer enables these particulars to be found. The three apertures determine a region of space containing all the useful light rays, and the instrument should have its optical elements of such sizes and in such positions that no ray which does not cross the boundary of this space is obstructed in its passage from the object through the instrument to the observer's eye. As a rule the space within which the optical elements of the instrument must be confined is specified, and the designer is called upon to produce an instrument satisfying these conditions and utilising only the minimum number of lenses. If the instrument is a terrestrial telescope his principal difficulty may be in reducing the overall length of the body to that desired. If a submarine periscope is in question the attainment of the specified length without either increasing the number of lenses or enlarging the diameter of the tube will involve special attention. Ability to find rapidly what is possible and to secure the most convenient principle of construction of the instrument is obviously important, and the present note proposes a simple method of dealing with the problem.

It is assumed that the apertures are circles lying in planes normal to the axis of symmetry of the instrument, and that their magnitudes and positions are known for either the object space or the image space. In the illustrations to be given it will be assumed that the various lenses between the objective and the eye lens may be permitted to have an aperture not exceeding a common value d. The modification necessary when the maximum aperture varies from lens to lens will be obvious.

Suppose then that the boundary to the channel containing the rays to be transmitted by the instrument is defined by means of three circular apertures which are met by a plane through the axis in P, P'; R, R'; S, S' (Fig. 3). The section of the boundary by this plane is the convex hexagon $RSP'R'S'P$, the sides SP' and $S'P$ passing through infinity. The image* of this hexagon formed by any part of the optical system of the instrument will likewise be a convex hexagon, and this

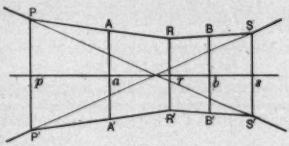


Fig. 3

* The reader can readily convince himself of the accuracy of the fundamental statements made in this paragraph if he bears in mind that the axis may be regarded as an infinite circle which is replaced after refraction by another circle derivable from the first by simple geometrical correspondence in which disarrangements of cyclical order are impossible. The only additional restriction is that the point where refraction takes place, that is the point in which the lens meets the axis, is common to both circles. By choosing any position for a lens and any axial point for a principal focus, and considering the images of p, r, s and of the points in which opposite sides of the hexagon meet (they are on the axis owing to the symmetry of the figure) the character of the transformed hexagon is determined. Gaussian imagery is assumed throughout the discussion.

image will determine the channel for the refracted rays. After every refraction there is necessarily some part of the boundary which extends to infinity, and this may correspond to any assigned normal section of the original channel. Only that part of the boundary which lies between successive lenses is of interest from the present point of view. These various segments pieced together and regarded as a whole constitute repeated outlines of the whole hexagon, which always presents its convex side to the axis, but the outline has an additional angle wherever a lens is encountered. At such places one section ends and another begins, and the angle at their junction is opposite in sign to the angles of the hexagon if the lens is positive. Lenses are introduced wherever necessary to bend back the boundary of the channel from encroaching over the prescribed bounds.

Fig. 4 illustrates the effects that may be produced by successive refractions, the heavy lines representing lenses. The various points of the original hexagon are identified by the same letters in their refracted images. The only limit to the amount of the initial hexagon that may be imaged between successive lenses arises from the fact that a pair of opposite sides of the hexagon always extend to infinity, and it is not possible in consequence to image one half of the outline on one side of the boundary within a single cell of the body.

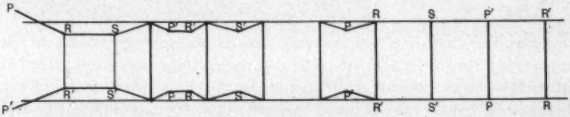


Fig. 4

The separation between the lenses limiting a given cell is readily found. Suppose that the lenses meet the boundary in planes corresponding to AA' and BB' of Fig. 3. If each of the transverse lines AA' and BB' has its image of diameter d, the transverse magnifications are $\frac{d}{AA'}$ and $\frac{d}{BB'}$. By a well-known elementary theorem the longitudinal magnification will be the product $\frac{d}{AA'}\cdot\frac{d}{BB'}$, and the separation of the lenses will thus be

$$\frac{d^2.ab}{AA'.BB'},$$

where a and b are the axial points of the planes AA' and BB'. The total length between the extreme lenses after any number of refractions is found by adding together the lengths corresponding to the successive intervals. An exceedingly important law is at once apparent, which has particular reference to instruments that are to be given the greatest attainable length. It is that this length will be proportional to the square of the diameter of the tube and to the number of lenses, and inversely proportional to the diameters of the field of view and of the external aperture of the system.

Particular cases which require special attention occur when the lenses occupy

positions corresponding to the limiting apertures PP', RR', SS'. If a lens is placed at every aperture image the greatest length is attained when the hexagon is developed into a straight line, so that the rays occupy the whole available space within the body of the instrument. The length reached is given by the sum

$$d^2\left\{\frac{sp}{SS'.P'P}+\frac{pr}{P'P.R'R}+\frac{rs}{R'R.S'S}+\frac{sp}{S'S.PP'}+\frac{pr}{PP'.RR'}+\ldots\right\}\ldots(1),$$

where it is assumed that SS' corresponds to the position of the first lens. All the terms in this sum necessarily have the same sign, as changes of sign take place simultaneously in the numerator and the denominator. Several alternative arrangements suggest themselves. Suppose for instance that a lens is placed at the image of every alternate aperture only. The length is then

$$d^2\left\{\frac{sr}{SS'.R'R}+\frac{rp}{R'R.PP'}+\frac{ps}{PP'.SS'}+\ldots\right\}\qquad\ldots.(2),$$

the terms having exactly the same values as before but differing in order. If in these cases the total number of lenses is a multiple of three the lengths of the two instruments will be exactly equal.

As another example suppose there were no field lenses, so that no lens is placed at any image of the aperture PP'. The sum is now

$$d^2\left\{\frac{sr}{SS'.R'R}+\frac{rs}{R'R.S'S}+\frac{sr}{S'S.RR'}+\ldots\right\}\qquad\ldots.(3),$$

every term being equal to one of the terms in (1). The other apertures may be treated in the same way, and it is evident that the first and second methods of construction will not give either the longest or the most compact instruments if the values of the individual terms differ. The estimation of the relative values of the terms is thus interesting, and this can be done by a very simple construction.

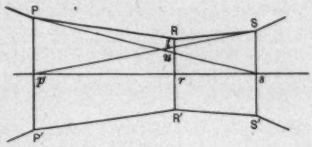


Fig. 5

Let pS and Ps meet RR' in t and u respectively (Fig. 5).

The relative values are in the ratios of the products

$$ps.RR' : pr.SS' : rs.PP',$$

that is of the triangles

$$pRs : pSr : rPs,$$

which are equal to the triangles

$$pRs : pts : pus,$$

which are in the ratio of their heights

$$Rr : tr : ur.$$

The length (1) may therefore be rewritten as

$$\frac{d^2.ps}{PP'.SS'}\left\{1+\frac{tr}{Rr}+\frac{ur}{Rr}+1+\frac{tr}{Rr}+\frac{ur}{Rr}+\ldots\right\}\qquad\ldots.(4).$$

PP' and SS' may be taken to represent the actual object and the entrance pupil, so that the factor outside the bracket will be known. The greatest attainable length of any cell not bounded at an image of the aperture which cuts down the marginal light has then a definite value. The next term gives the length of a cell bounded by an image of the field and of the intermediate stop. As ps may be always considered large in comparison with rs, the utmost that can be done is to make this cell equal in length to the previous one by making the diameter of the intermediate stop image of the same size as the entrance pupil. This involves an immediate fall in the light intensity as soon as the object departs from the axis. The length of the third cell, which is bounded by images of the intermediate stop and of the entrance pupil, depends upon the relative illumination at the edge and at the centre of the field of view. If the width of the marginal beam in the axial plane of symmetry is half that of the central beam, PRs will be a straight line, and the lengths of the first and third cells will be equal. If the edge illumination is increased, the length of this cell is diminished, and if the illumination falls the length may be increased. When the width of the marginal beam is greater than half the central width, it will evidently be possible to attain a greater length per lens when all field lenses are omitted than when they are present.

The case when all three cells are of equal length is a convenient one to take as a standard of reference, and systems of this type possess great flexibility. It is always possible to attain a given standard extent of field with a standard aperture and the distribution of illumination characteristic of the type with any given number of cells. The length per cell thus gives complete information about such systems. This may be illustrated by the following examples. The cells begin with the first objective, indicated by S, and end with the formation of a real upright image of the object, denoted by P. If the system ended with P' the final image would be inverted.

Table I.

Number of cells	Lens positions	Remarks
2	S, R', P	Not divisible into separate telescopes
3	S, P', S', P	" " " "
3	S, P', R', P	" " " "
3	S, R', S', P	No field lens
4	S, P', R', S', P	
5	S, R', P, S, R', P	Not divisible into separate telescopes
6	15 solutions	A lens coincides with at least one image of each stop
7	35 solutions	At least two lenses coincide with images of each stop

In general with $2n$ inversions and p cells the number of solutions is

$$\frac{p!}{(6n-p-2)!\,(2p+2-6n)!},$$

where p may have any value from $3n-1$ to $6n-2$.

It does not follow that because several solutions enable the same length of body to be obtained with the same number of lenses that these are all equally desirable. Other things being equally satisfactory the solution preferred will be that which involves the use of the least powerful lenses. It should be immediately evident that the omission to place a lens where an image of a certain aperture is formed will involve increases in the powers of the lenses at the two neighbouring images, since the deviations at the margins of these lenses must be greater. The failure therefore to fill completely the space available for the rays will be associated with increased lens powers and consequently with greater trouble due to secondary colour effects and increased curvature of field. To determine how serious these defects are likely to be the powers of the lenses are required. There is no need to consider the refracted light to determine these powers, but attention may be directed as before to the object space hexagon. If for example successive lenses are to be located at images of PP', RR', SS', the lens at the image of RR' is required to deviate the image of RS into alignment with the preceding image of PR. Now the angular magnification at any plane is inversely proportional to the linear magnification, and the angular deviation is equal to the product of the lens power and the distance from the axis at which refraction takes place. It at once follows that the power of the lens required is $\left(\frac{RR'}{d}\right)^2$ times the power of the lens which would produce the corresponding effect in the object space boundary. This is in reality the same result that has already been reached regarding the length of the instrument, since the focal lengths of the lenses evidently bear a fixed relation to the lengths of the cells, but it serves to show how great is the advantage from every point of view in making the diameter of the body as great as possible. Various expressions are easily found for the power of the lens necessary at RR'. Let this be denoted by $\kappa \left(\frac{d}{RR'}\right)^2$, so that κ is the power of the lens required in the instrument. The deviation at R is then $\frac{1}{2}\frac{\kappa d^2}{\overline{RR'}}$, and the area of the triangle PRS by various expressions leads to the equations, when inclinations to the axis are assumed to be small,

$$\frac{\kappa d^2}{\overline{RR'}}.\overline{pr}.\overline{rs} = 2\rho R.\overline{ps} \qquad \ldots.(5)$$

$$= \overline{PP'}.\overline{rs} + \overline{RR'}.\overline{sp} + \overline{SS'}.\overline{pr} \qquad \ldots.(6),$$

where ρ is the point of intersection of Rr and PS. Equation (6) itself shows the result reached above in a different way that κd^2 is unaltered by refraction. For it may be rewritten in the form

$$\kappa d^2 = \frac{\frac{\overline{rs}}{\overline{RR'}.\overline{SS'}} + \frac{\overline{sp}}{\overline{SS'}.\overline{PP'}} + \frac{\overline{pr}}{\overline{PP'}.\overline{RR'}}}{\frac{\overline{pr}}{\overline{PP'}.\overline{RR'}}\cdot\frac{\overline{rs}}{\overline{RR'}.\overline{SS'}}},$$

and each term on the right in invariant on refraction. The expressions of this

type for the powers of the lenses at images of PP', RR' and SS' when no images are without lenses are evidently

$$\left.\begin{aligned} d^2\kappa_{RSP'} &= d^2\kappa_{R'S'P} = SS'\,\frac{RR'.sp + SS'.pr + P'P.rs}{rs.sp} \\ d^2\kappa_{S'PR} &= d^2\kappa_{SP'R'} = P'P\,\frac{SS'.pr + P'P.rs + R'R.sp}{sp.pr} \\ d^2\kappa_{PRS} &= d^2\kappa_{P'R'S'} = RR'\,\frac{PP'.rs + RR'.sp + SS'.pr}{pr.rs} \end{aligned}\right\} \quad \ldots.(7),$$

where all the factors are considered with due regard to sign. In each numerator two terms have one sign and the third the opposite sign, in each case giving rise to a positive κ. In the standard case the terms are all equal and the focal length of each lens is equal to the length of a cell. This is readily seen geometrically by considering the intercepts on the third aperture by the line forming the boundary between the other two.

The cases in which an image of an aperture is without a lens may be treated by analogy with (7).

$$\left.\begin{aligned} d^2\kappa_{RSR'} &= d^2\kappa_{R'S'R} = d^2\kappa_{S'RS} = d^2\kappa_{SR'S'} = \frac{2RR'.SS'}{rs} \\ d^2\kappa_{SP'S'} &= d^2\kappa_{S'PS} = d^2\kappa_{PSP'} = d^2\kappa_{P'S'P} = \frac{2PP'.SS'}{ps} \\ d^2\kappa_{PRP'} &= d^2\kappa_{P'R'P} = d^2\kappa_{R'PR} = d^2\kappa_{RP'R'} = \frac{2PP'.RR'}{pr} \end{aligned}\right\} \quad \ldots.(8),$$

showing that in the standard case the omission of a lens requires the power of each neighbouring lens to be doubled. Again if a lens is omitted on each side of a given lens expressions are obtained corresponding to (7) but with every term of the same sign, so that in the standard case the power of the lens is three times its previous value. The expressions are

$$\left.\begin{aligned} d^2\kappa_{PSR'} &= d^2\kappa_{P'S'R} = SS'\,\frac{PP'.sr + SS'.rp + R'R.ps}{rs.sp} \\ d^2\kappa_{R'PS} &= d^2\kappa_{RP'S'} = PP'\,\frac{R'R.ps + PP'.sr + SS'.rp}{sp.pr} \\ d^2\kappa_{SR'P} &= d^2\kappa_{S'RP'} = R'R\,\frac{SS'.rp + R'R.ps + PP'.sr}{pr.rs} \end{aligned}\right\} \quad \ldots.(9).$$

By considering the various cases that may arise it is seen that the effect of omitting to place a lens at an aperture image is to contribute for this vacant aperture twice the normal contribution to the sum of the powers of the lenses, assuming that the standard case is under consideration. If account is taken of a final field lens but not of an eye lens, with $2n$ inversions and p cells the sum of the powers will be proportional to $12n - p - 3$. In this respect all arrangements which involve the same number of lenses and the same number of inversions are on an equality in the standard case.

In the general case the increase in power associated with the omission of a lens at an image of P is

$$-\frac{(SS'.pr + P'P.rs + R'R.sp)^2}{d^2.pr.rs.sp} \quad \ldots\ldots(10),$$

the term in the bracket being that in the numerator of the fraction in the second of equations (7). The corresponding expressions for other cases are easily written down. The sum of the additions incurred by each separate omission gives the total addition to be made to the sum of the powers.

A convenient expression for the increment in power in terms of the lengths of the cells is

$$\frac{\kappa^2 l_1 l_2}{l_3} \quad \ldots\ldots(11),$$

where κ is the power of the lens which it is proposed to omit, l_1 and l_2 are the lengths of the cells on either side, and l_3 is the length of the combined cell when the omission is made. The three lengths l_1, l_2, l_3 are those considered earlier in this discussion when the length attainable by a periscope was investigated.

It is evident that when weight is attached both to the attainment of great length and to a minimum value of the power sum the most favourable type of construction is that which has been generally adopted for submarine periscopes, with a lens at every image of the three apertures.

In investigating the attainment of great length attention was directed to each cell as a separate element. It has still to be considered whether, when the conditions permit of it, any advantage will be gained by deliberately introducing inequalities in the lengths of the three cells. Suppose the lengths of the cells exceed the mean value L by amounts $\lambda_1, \lambda_2, \lambda_3$, where $\lambda_1 + \lambda_2 + \lambda_3 = 0$. From (7)

$$\kappa_1 = \frac{l_2 + l_3 - l_1}{l_2 l_3},$$

with symmetrical expressions for κ_2 and κ_3, so that

$$\kappa_1 + \kappa_2 + \kappa_3 = \frac{3L^2 - 2(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)}{L^3 - \frac{1}{2}L(\lambda_1^2 + \lambda_2^2 + \lambda_3^2) + \lambda_1\lambda_2\lambda_3}$$

$$= \frac{3}{L} - \frac{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}{2L^3 - L(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)} \quad \text{approximately,}$$

showing that inequalities in length tend to reduce the aberrations which depend on the sum of the powers. In this respect the standard form is the least satisfactory solution of its class, but the advantage to be gained by uneven spacing is small, since it is a second order effect. For instance if the lengths are in the ratios 2:3:4 the reduction in the power sum is slightly more than 4 per cent.

It has hitherto been assumed that the lenses will be placed in the planes at which aperture images are formed. In the case of field lenses exact coincidence with the image is generally undesirable, and the effect of placing the lenses in other planes requires consideration. Obviously the sizes of the lenses are determined by the sections of the channel to which their positions correspond. Suppose

in Fig. 3 the planes AA' and BB' are proposed as lens planes. The relation between transverse and longitudinal magnitudes is

$$AA' . pr = PP' . ar + RR' . pa,$$

and thus if the apertures PP', RR', SS' relate to a standard form

$$\begin{aligned} AA' . SS' . pr &= (ar . ps + pa . rs) \frac{d^2}{l} \\ &= as . pr . \frac{d^2}{l}, \end{aligned}$$

showing that a cell bounded by AA' and SS' will be of standard length. Similarly

$$PP' . BB' = pb . \frac{d^2}{l},$$

and combining the two cases

$$AA' . BB' . ps = pb . as . \frac{d^2}{l}.$$

Thus if l' is the length of the cell bounded by AA' and BB'

$$\frac{l'}{l} = \frac{ab . ps}{pb . as},$$

or

$$\frac{l - l'}{l} = \frac{pa . bs}{pb . as},$$

showing that if two successive lenses are not in planes corresponding to the angular points of the hexagon the length of the cell they bound is reduced.

By considering the triangle ARS it is readily seen that as AA' moves from PP' to RR' the deviations necessary at A and S continually increase, so that the powers of the lenses increase. It is to be expected in view of the result reached earlier regarding the effect of omitting a lens on the power, that the introduction of an additional lens will reduce the power sum for a given length. This is very simply investigated for the case when the powers of the lenses are all equal and the cell lengths are equal. The connection between the length of a cell and the power of a single lens with 2, 3, 4 and 5 lenses to an inversion is given in Table II. If l

Table II.

Number of lenses	Value of κl
2	2
3	1
4	$2 - \sqrt{2}$
5	$\frac{1}{2}(3 - \sqrt{5})$

is given the same value in all cases the total power involved per inversion falls, but the effective aperture is reduced. To obviate this by giving κ the value suitable for three lenses would lead, for a greater number of lenses, to conditions worse from every point of view. Between these two extremes new types of construction may be reached which yield more satisfactory results than those hitherto employed in periscopes.

The discussion of telescopes in which an attempt is made to secure a short body with a small value for the power sum involves the consideration of a compromise between conditions which tend to be in direct conflict. This compromise must be based upon a knowledge of aberrations, and the discussion therefore lies beyond the scope of this paper.